Worksheet 3.1—Limits

Show all work. Unless stated otherwise, no calculator permitted.

1. Explain in your own words what is meant by the equation $\lim_{x\to 2} f(x) = 4$. Is it possible for this statement to be true and yet f(2) = 5? Explain. What graphical feature would be manifested in this situation?

2. Explain what it means to say that $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 6$.

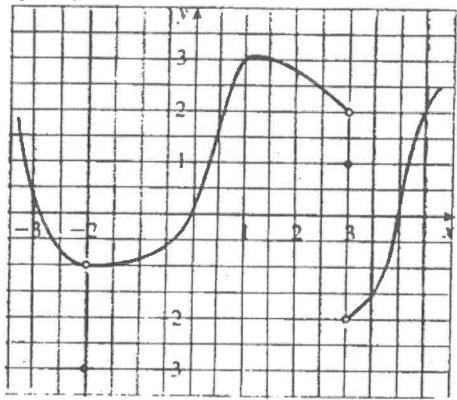
In this situation, is it possible that $\lim_{x\to 1} f(x)$ exists? Explain. What graphical feature would be manifested in this situation?

3. Explain the meaning of each of the following, then sketch a graph of a function exhibiting the indicated behavior.

(a)
$$\lim_{x \to -2} f(x) = \infty$$

(b)
$$\lim_{x \to -3^+} g(x) = -\infty$$
.

4. For the function f whose graph is given at below, state the value of the given quantity, if it exists. If it does not exist, explain why.



(a)
$$\lim_{x \to 1} f(x) =$$

(b)
$$\lim_{x \to \infty} f(x) =$$

(a)
$$\lim_{x \to 1} f(x) =$$
 (b) $\lim_{x \to 3^{-}} f(x) =$ (c) $\lim_{x \to 3^{+}} f(x) =$ (d) $\lim_{x \to 3} f(x) =$

(d)
$$\lim_{x \to 3} f(x) =$$

(e)
$$f(3) =$$

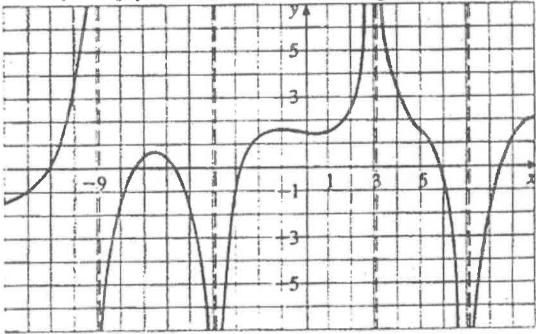
(f)
$$\lim_{x \to -2^-} f(x) =$$

(e)
$$f(3) =$$
 (f) $\lim_{x \to -2^{-}} f(x) =$ (g) $\lim_{x \to -2^{+}} f(x) =$ (h) $\lim_{x \to -2} f(x) =$ (i) $f(-2) =$

(h)
$$\lim_{x \to \infty} f(x) =$$

(i)
$$f(-2) =$$

5. For the function f whose graph is shown at below, state the following.



(a)
$$\lim_{x \to 3} f(x) =$$

(b)
$$\lim_{x \to 7} f(x) =$$

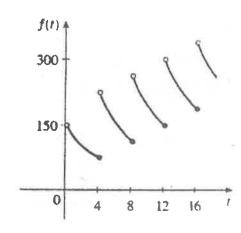
(c)
$$\lim_{x \to -4} f(x) =$$

(d)
$$\lim_{x \to -9^-} f(x) =$$

(e)
$$\lim_{x \to -9^+} f(x) =$$
 (f) $\lim_{x \to -9} f(x) =$

(f)
$$\lim_{x \to -9} f(x) =$$

- (g) The equations of the vertical asymptotes
- 6. A patient receives a 150-mg injection of a drug every four hours. The graph at right shows the amount f(t) of the drug in the bloodstream after t hours. Find $\lim_{t\to 12^-}f(t)$ and $\lim_{t\to 12^+}f(t)$ and then explain the significance/meaning of these one-sided limits in terms of the injections.



7. (Calculator Permitted) Sketch the graph of the function $f(x) = \frac{1}{1+2^{1/x}}$ and state the value of each

limit, if it exists. If it does not exist, explain why. State any discontinuities.

(a)
$$\lim_{x \to 0^-} f(x) =$$

(b)
$$\lim_{x \to 0^+} f(x) =$$

(c)
$$\lim_{x\to 0} f(x) =$$

(d)
$$f(0) =$$

8. Sketch the graph of the following function, and use it to help you determine the values of a for which $\lim f(x)$ exists. Describe any discontinuities.

$$f(x) = \begin{cases} 2 - x, & x < -1 \\ x_2 & -1 \le x < 1 \\ (x - 1)^2, & x \ge 1 \end{cases}$$

9. (Calculator permitted) Fill in the table for the following function, then use the numerical evidence to evaluate the indicated limit. (Be sure you're in radian mode)

$$f(x) = \frac{\sin(3x)}{x} =$$

x	-0.1	-0.01	0	0.001	0.01	0.1
f(x)						
, (")						

10. (Calculator permitted) Fill in the table for the following function, then use the numerical evidence to evaluate the indicated limit.

$$f(x) = \frac{1 - \cos x}{x^2} =$$

0 0.001	0.01 0.1
	0 0.003