

Calculus

name: Key

Exercises: Evaluate each limit by direct substitution and/or algebraic simplification. Then graph each problem with any holes or asymptotes.

1) $\lim_{x \rightarrow 3} x^2 = 3^2 = 9$
 no hole
 no asymptote

2) $\lim_{x \rightarrow -5} \left(\frac{x^2 + 25}{x + 5} \right) = \frac{25 + 25}{-5 + 5} = \frac{50}{0}$ Asymptote @ $x = -5$
 $\lim_{x \rightarrow -5} f(x) = \boxed{\text{DNE}}$

3) $\lim_{x \rightarrow -4} \left(\frac{6x}{x-2} \right) = \frac{6(-4)}{-4-2} = \frac{-24}{-6} = 4$
 Asymptote @ $x = 2$

4) $\lim_{x \rightarrow 0} \left(\frac{1}{x} \right) =$ asymptote @ $x = 0$
 $\lim_{x \rightarrow 0} f(x) = \boxed{\text{DNE}}$

5) $\lim_{x \rightarrow 5} \left(\frac{x^2 - 25}{x - 5} \right) = \lim_{x \rightarrow 5} \frac{(x-5)(x+5)}{x-5} = \lim_{x \rightarrow 5} x + 5 = 5 + 5 = 10$
 hole @ $x = 5$ (5/10)

6) $\lim_{x \rightarrow 6} \left(\frac{x^2 - 25}{x - 5} \right) = \frac{6^2 - 25}{6 - 5} = \frac{36 - 25}{1} = 11$
 hole @ $x = 5$ (5/10)

7) $\lim_{x \rightarrow 2} \left(\frac{x^3 - 8}{x - 2} \right) = \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{x-2} = \lim_{x \rightarrow 2} x^2 + 2x + 4 = 2^2 + 2(2) + 4 = 12$
 hole @ $x = 2$ (2, 12)

8) $\lim_{x \rightarrow -3} \left(\frac{x^3 + 27}{x + 3} \right) = \lim_{x \rightarrow -3} \frac{(x+3)(x^2 - 3x + 9)}{x+3} = \lim_{x \rightarrow -3} x^2 - 3x + 9 = (-3)^2 - 3(-3) + 9 = 9 + 9 + 9 = 27$
 hole @ $x = -3$ (-3, 27)

9) $\lim_{x \rightarrow -1} \left(\frac{2x^2 + 3x + 1}{x + 1} \right) = \lim_{x \rightarrow -1} \frac{(2x+1)(x+1)}{x+1} = \lim_{x \rightarrow -1} 2x + 1 = 2(-1) + 1 = -1$
 hole @ $(-1, -1)$

10) $\lim_{x \rightarrow -1} \left(\frac{x+1}{2x^2 + 3x + 1} \right) = \lim_{x \rightarrow -1} \frac{x+1}{(2x+1)(x+1)} = \lim_{x \rightarrow -1} \frac{1}{2x+1} = \frac{1}{-1} = -1$
 hole @ $(-1, -1)$
 Asymptote @ $x = -1/2$

11) $\lim_{x \rightarrow -1/2} \left(\frac{x+1}{2x^2 + 3x + 1} \right) =$
 $\boxed{\text{DNE}}$
 Asymptote @ $x = -1/2$
 hole @ $(-1, -1)$

12) $\lim_{x \rightarrow 0} \left(\frac{x^2 + 25}{x + 5} \right) = \frac{0 + 25}{0 + 5} = \frac{25}{5} = 5$
 Asymptote @ $x = -5$