

Name _____

Key

Date _____

Period _____

—Properties of Limits

Show all work. Unless stated otherwise, no calculator permitted.

Short Answer

1. Given that $\lim_{x \rightarrow a} f(x) = -3$, $\lim_{x \rightarrow a} g(x) = 0$, $\lim_{x \rightarrow a} h(x) = 8$, for some constant a , find the limits that exist.

If the limit does not exist, explain why.

(a) $\lim_{x \rightarrow a} [f(x) + h(x)] =$

(b) $\lim_{x \rightarrow a} [f(x)]^2 =$

(c) $\lim_{x \rightarrow a} \sqrt[3]{h(x)} =$

(d) $\lim_{x \rightarrow a} \frac{1}{f(x)} =$

$$\begin{aligned}
 &= \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} h(x) = [\lim_{x \rightarrow a} f(x)]^2 \quad \sqrt[3]{\lim_{x \rightarrow a} h(x)} \\
 &= -3 + 8 = [-3]^2 \quad = \sqrt[3]{8} \\
 &= \boxed{5} \quad = \boxed{9} \quad = \boxed{2} \\
 &\quad = \frac{1}{\lim_{x \rightarrow a} f(x)} \\
 &\quad = \frac{1}{-3} \\
 &\quad = \boxed{-\frac{1}{3}}
 \end{aligned}$$

(e) $\lim_{x \rightarrow a} \frac{f(x)}{h(x)} =$

(f) $\lim_{x \rightarrow a} \frac{g(x)}{f(x)} =$

(g) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} =$

(h) $\lim_{x \rightarrow a} \frac{2f(x)}{h(x) - f(x)} =$

$$\begin{aligned}
 &= \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} h(x)} \\
 &= \frac{-3}{8} \\
 &= \boxed{-\frac{3}{8}}
 \end{aligned}$$

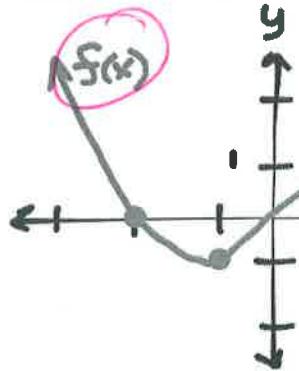
$$\begin{aligned}
 &= \frac{\lim_{x \rightarrow a} g(x)}{\lim_{x \rightarrow a} f(x)} \\
 &= \frac{0}{-3} \\
 &= \boxed{0}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \\
 &= \frac{-3}{0} \\
 &= \boxed{\text{DNE}}
 \end{aligned}$$

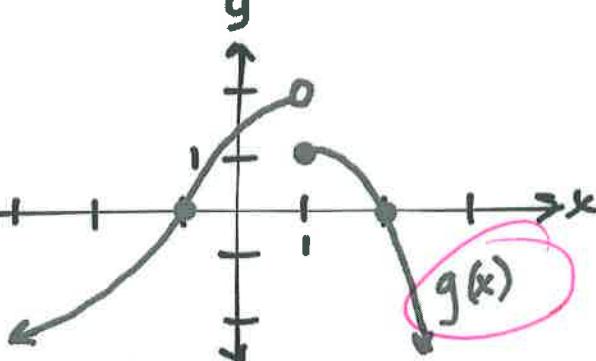
$$\begin{aligned}
 &= \frac{2 \lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} h(x) - \lim_{x \rightarrow a} f(x)} \\
 &= \frac{2(-3)}{8 - (-3)} \\
 &= \boxed{-\frac{6}{11}}
 \end{aligned}$$

2. The graphs of f and g are given below. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.

$f(x)$



y



$$(a) \lim_{x \rightarrow 2} [f(x) + g(x)] =$$

$$(b) \lim_{x \rightarrow 1} [2f(x) - 3g(x)] =$$

$$(c) \lim_{x \rightarrow 0} [f(x)g(x)] =$$

$$\begin{aligned} \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} g(x) &= 2 \lim_{x \rightarrow 1} f(x) - 3 \lim_{x \rightarrow 1} g(x) \\ &= (\lim_{x \rightarrow 0} f(x)) \cdot (\lim_{x \rightarrow 0} g(x)) \end{aligned}$$

$$= 2 + 0$$

$$= 2(1) - 3(\text{DNE})$$

$$= \boxed{\text{DNE}}$$

$$= 0(1.4)$$

$$= \boxed{0}$$

$$(d) \lim_{x \rightarrow -1} \frac{f(x)}{g(x)} =$$

$$(e) \lim_{x \rightarrow 2} x^3 f(x) =$$

$$(f) \lim_{x \rightarrow 1^-} f(g(x)) =$$

$$= \frac{\lim_{x \rightarrow -1} f(x)}{\lim_{x \rightarrow -1} g(x)}$$

$$= \frac{-1}{0}$$

= DNE

$$= \lim_{x \rightarrow 2} x^3 \cdot \lim_{x \rightarrow 2} f(x)$$

$$= 2^3 \cdot 2$$

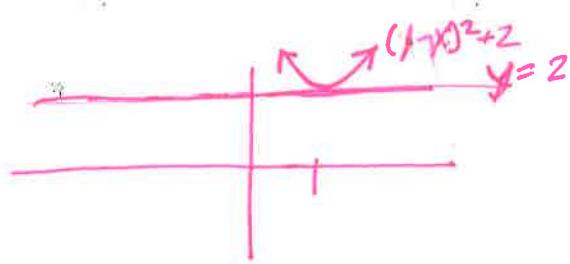
$$= 8 \cdot 2$$

$$= \boxed{16}$$

$$f\left(\lim_{x \rightarrow 1^-} g(x)\right)$$

$$= \lim_{x \rightarrow 2^-} f(x)$$

$$= \boxed{2}$$

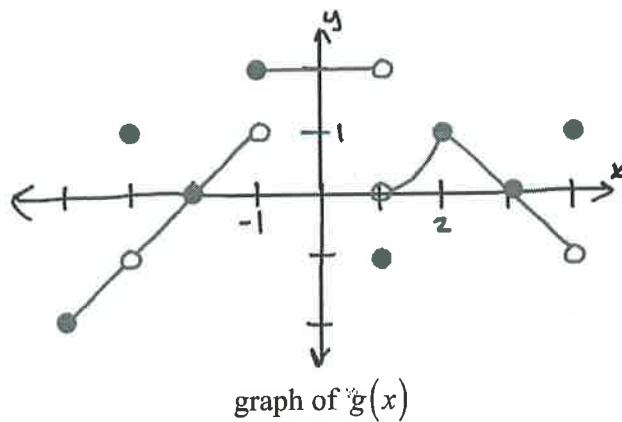
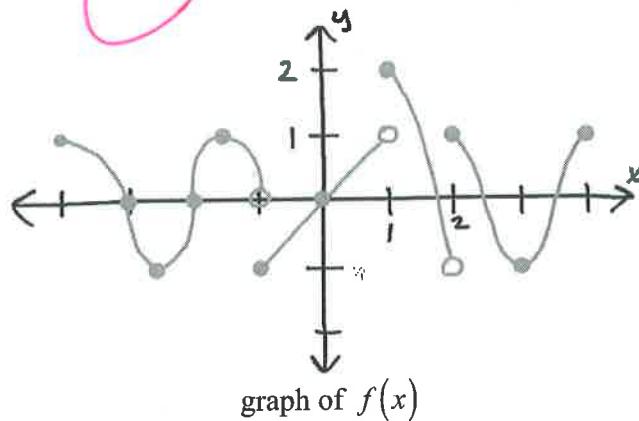


Multiple Choice

Squeeze theorem

- B 6. Suppose $2 \leq f(x) \leq (1-x)^2 + 2$ for all $x \neq 1$ and that $f(1)$ is undefined. What is $\lim_{x \rightarrow 1} f(x)$?

- (A) 3 (B) 2 (C) 4 (D) $\frac{5}{2}$ (E) 1



Use the graphs of the function $f(x)$ and $g(x)$ shown above to answer questions 7 – 9.

B 7. $\lim_{x \rightarrow 2^-} \left(\frac{f(x)}{g(x)} \right) = \frac{-1}{1} = -1$

- (A) 1 (B) -1 (C) 2 (D) -2 (E) DNE

A 8. $\lim_{x \rightarrow -3^+} f(g(x)) = \lim_{x \rightarrow -3^+} f(x) = 0$

- (A) 0 (B) -1 (C) 2 (D) 1 (E) DNE

A 9. $g(1) + \lim_{x \rightarrow -1^+} x \cdot f(x) = -1 + \left(\lim_{x \rightarrow -1^+} x \cdot \lim_{x \rightarrow -1^+} f(x) \right)$

- (A) 0 (B) -1 (C) 2 (D) 1 (E) DNE

$$\begin{aligned}
 &= -1 + (-1) \cdot (-1) \\
 &= -1 + 1 \\
 &= 0
 \end{aligned}$$