Answer Key

1. Find $\lim_{x\to 0}\frac{\sin(x)}{x}$, by using squeeze theorem with the bounds of cos(x) ≤ sin(x)/x ≤ 1 . Use a graph to model your answer.

Answer = 1

2. If $8\cos\left(\frac{\pi x}{6}\right) \le g(x) \le \frac{-2\pi}{\sqrt{3}}(x-2)+4$ for all x near 2 except perhaps at x=2 itself, what is the value of $\lim_{x\to 2} g(x)$?

Answer:

1 / 4 Let $f(x) = 8\cos\frac{\pi x}{6}$ and $h(x) = \frac{-2\pi}{\sqrt{3}}(x-2)+4$. We know that both f and h are continuous functions which means that the limits of f and h exist at all x.

2 / 4 We are given that $8\cos\frac{\pi x}{6} \le g(x) \le \frac{-2\pi}{\sqrt{3}}(x-2)+4$ for all x near 2 except perhaps at $x = 2$ itself, so it follows that

$$f(x) \le g(x) \le h(x) \text{ for } x \text{ near } 2.$$

This means that

$$\lim_{x\to 2} f(x) \le \lim_{x\to 2} g(x) \le \lim_{x\to 2} h(x)$$

provided the limits exist.

3 / 4 We substitute and get

$$\lim_{x\to 2} 8\cos\frac{\pi x}{6} \le \lim_{x\to 2} g(x) \le \lim_{x\to 2}\left(\frac{-2\pi}{\sqrt{3}}(x-2)+4\right).$$

Evaluating the limit on the left and the limit on the right gives

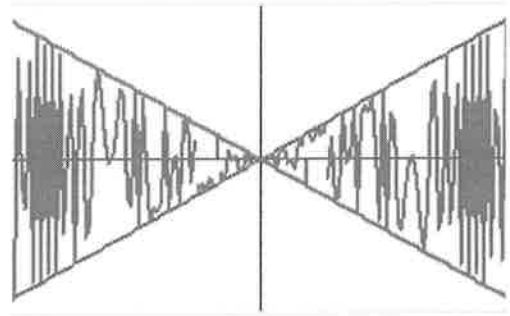
$$4 \le \lim_{x\to 2} g(x) \le 4.$$

4 / 4 By the squeeze theorem,

$$\lim_{x\to 2} g(x) = 4.$$

3. The graphs of the functions $f(x)=x$, $g(x)=-x$, and $h(x)=x\cos\left(\frac{50\pi}{x}\right)$ on the interval $-1\le x\le 1$ are given at right.

Use the Squeeze Theorem to find $\lim_{x\to 0} x\cos\left(\frac{50\pi}{x}\right)$. Justify.



$\lim_{x\to 0} f(x) = 0 = \lim_{x\to 0} g(x)$

and since either $-|x| \le h(x) \le |x|$ or $x \le h(x) \le -x$ for $x<0$ and $-x \le h(x) \le x$ for $x>0$

then, by the Squeeze Theorem,

$\lim_{x\to 0} x\cos\left(\frac{50\pi}{x}\right) = 0$ too!

4. If $1\le f(x)\le x^2+2x+2$ for all x, find $\lim_{x\to -1} f(x)$. Justify.

$1 \le f(x) \le x^2+2x+2$

$(-1)^2+2(-1)+2$
$=1-2+2=1$

$\lim_{x\to -1} 1 \le \lim_{x\to -1} f(x) \le \lim_{x\to -1} x^2+2x+1$

by Squeeze theorem,

$1 \le \boxed{1} \le 1$

$\lim_{x\to -1} f(x) = \boxed{1}$

5. If $-3\cos(\pi x)\le f(x)\le x^3+2$, evaluate $\lim_{x\to 1} f(x)$. Justify

$\lim_{x\to 1}(-3\cos(\pi x))$ $\lim_{x\to 1}(x^3+2)$

$-3\cos\pi$ 3

3

$\lim_{x\to 1} -3\cos(\pi x) \le \lim_{x\to 1} f(x) \le \lim_{x\to 1} x^3+2$

Since $-3\cos(\pi x) \le f(x) \le x^3+2$,

by the Squeeze theorem,

$\lim_{x\to 1} f(x) = 3$ too!