CALCULUS

WORKSHEET ON CONTINUITY AND INTERMEDIATE VALUE THEOREM

Work the following on **notebook paper**.

On problems 1 – 4, sketch the graph of a function f that satisfies the stated conditions.

1. f has a limit at $x=3$, but it is not continuous at $x=3$.

2. f is not continuous at $x=3$, but if its value at $x=3$ is changed from $f(3)=1$ to $f(3)=0$, it becomes continuous at $x=3$.

3. f has a removable discontinuity at $x=c$ for which $f(c)$ is undefined.

4. f has a removable discontinuity at $x=c$ for which $f(c)$ is defined.

On problems 5 – 7, use the definition of continuity to prove that the function is discontinuous at the given value of a. Sketch the graph of the function.

5. $f(x)=\dfrac{x^2-5x+4}{x-1}$ $\quad a=1$

6. $g(x)=\begin{cases} \dfrac{x^2-3x}{x^2-9} & \text{if } x\neq 3 \\ 1 & \text{if } x=3 \end{cases}$ $\quad a=3$

7. $h(x)=\begin{cases} e^x & \text{if } x<0 \\ x^2 & \text{if } x\geq 0 \end{cases}$ $\quad a=0$

On problems 8 – 11, use the definition of continuity to find the values of k and/or m that will make the function continuous everywhere.

8. $f(x)=\begin{cases} kx^2, & x\leq 2 \\ 2x+k, & x>2 \end{cases}$

9. $g(x)=\begin{cases} x^2+5, & x>2 \\ m(x+3)+k, & -1<x\leq 2 \\ 2x^3+x+7, & x\leq -1 \end{cases}$

10. $h(x)=\begin{cases} \dfrac{x^2-k^2}{x-k}, & x\neq k \\ 8, & x=k \end{cases}$

11. $p(x)=\begin{cases} \dfrac{4\sin x}{x}, & x<0 \\ m-2x-5, & x\geq 0 \end{cases}$

On problems 12 – 14, a function f and a closed interval $[a,b]$ are given. Show whether the conditions of the Intermediate Value Theorem hold for the given value of k. If the conditions hold, find a number c such that $f(c)=k$. If the theorem does not hold, give the reason. Whether the theorem holds or not, sketch the curve and the line $y=k$.

12. $f(x)=2+x-x^2$

$[a,b]=[0,3]$

$k=1$

13. $f(x)=\sqrt{25-x^2}$

$[a,b]=[-4.5,3]$

$k=3$

14. $f(x)=\dfrac{1}{x-2}$

$[a,b]=[3,5]$

$k=\dfrac{5}{6}$

TURN->>>

15. Use the Intermediate Value Theorem to show that $f(x)=x^3+x$ takes on the value 9 for some x in [1, 2].

16. Show that $g(t)=\frac{t}{t+1}$ takes on the value 0.499 for some t in [0, 1].

17. Show that $f(x)=\frac{x^2}{x^7+1}$ takes on the value 0.4

On problems 18 – 19, use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

18. $x^4+x-3=0$ $\quad [1, 2]$

19. $\cos x = x$ $\quad \left[0, \frac{\pi}{2}\right]$

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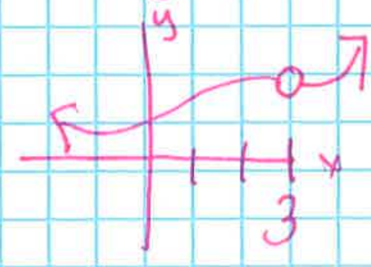
20. In 1996 the United States postage rate for a first class letter was \$0.32 for the first ounce and \$0.23 per ounce thereafter. Does the function meet the hypotheses of the Intermediate Value Theorem? Is there a weight of letter that can be mailed for exactly \$1.00? Justify your answers.

21. Jesse and Kay ran a 1000-m race. One minute after the race began, Jesse was running 20 km/hr, and Kay was running 15 km/hr. Three minutes after the race began, Jesse has slowed to 17km/hr, and Kay had speeded up to 19 km/hr. Assume that each runner's speed is a continuous function of time. Prove that there is a time between 1 minute and 3 minutes after the race began at which each one was running exactly the same speed. Is it possible to tell what that speed is? Is it possible to tell when that speed occurred? Explain.

Key

1)



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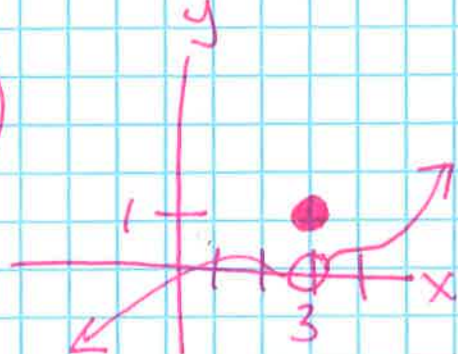

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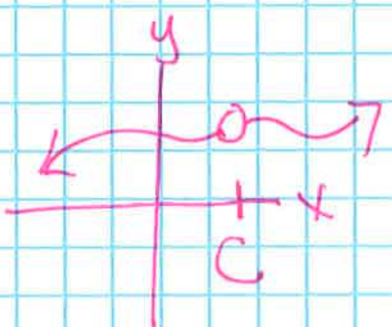
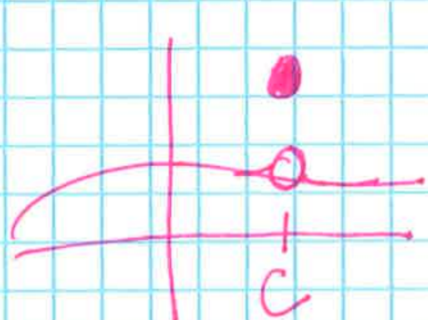


4)



5) $\frac{(1)^2-5(1)+4}{1-1} = \frac{0}{0}$ hole @ a=1

6) $\frac{3^2-3(3)}{3^2-9} = \frac{0}{0}$ hole

$\lim_{x\to 3} \frac{x^2-3x}{x^2-9} = \lim_{x\to 3} \frac{x\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}$

$= \lim_{x\to 3} \frac{x}{x+3} = \frac{3}{3+3} = \frac{3}{6} = \boxed{\frac{1}{2}}$

$\frac{1}{2} \neq 1$

7) $\lim_{x\to 0} e^x = e^0 = 1$

$\lim_{x\to 0} x^2 = 0^2 = 0$

not equal @ x=0

8) $K(2)^2 = 4K$

$2(2)+K = K+4$

$4K = K+4$
$-K \quad -K$

$3K = 4$

$\boxed{K = 4/3}$

9) $(2)^2+5 = 4+5 = 9$

$m(2+3)+K$

$\boxed{5m+K=9}$

$2(-1)^3+(-1)+7$
$-2-1+7$
$=4$

$m(-1+3)+K$

$\boxed{2m+K=4}$

$5m+K=9$
$-2m-K=-4$
$3m=5$ $\quad m=5/3$

$2(5/3)+K=4$
$10/3+K=4$
$-10/3 \quad -10/3$
$K=2/3$

10) $\frac{K^2-K^2}{K-K}=\frac{0}{0}$ hole

$$\lim_{x\to K}\frac{\cancel{(x-K)}(x+K)}{\cancel{x-K}}=K+K=2K$$

$2K=8$

$\boxed{K=4}$

11) $m-2(0)-5$

$m-5$

$\lim_{x\to 0}\frac{4\sin x}{x}$ by graph $=4$

$m-5=4$

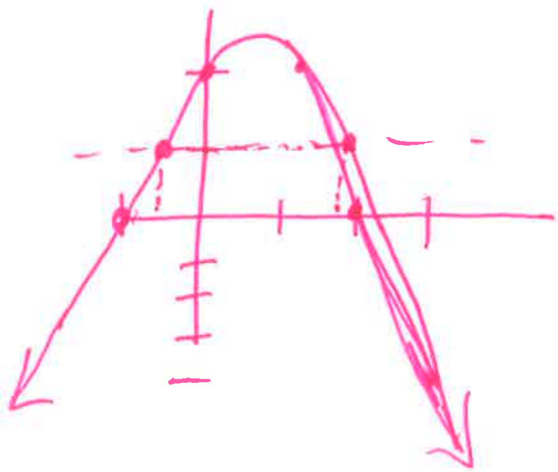
$\boxed{m=9}$

12) $x=?$

$y=1$

$f(0)=2$

$f(3)=2+3-9=-4$



$-4<y<2$

$0<x<3$

$1=2+x-x^2$

$-1 \quad -1$

$-x^2+x+1=0$

$\cancel{-(x^2-x-1)=0}$

$\cancel{-(\ \ )(\ \ )}$

$\frac{-1\pm\sqrt{1^2-4(-1)(2)}}{2(-1)}$

$\frac{+1\pm\sqrt{5}}{+2}$

$\frac{1\pm\sqrt{5}}{2}$

1.618 in bound

-.618 out of bound

$\cancel{\frac{1\pm\sqrt{5}}{2}=\frac{3}{2}}$

$\cancel{\frac{1-\sqrt{5}}{2}=\boxed{-1}}$

13) $\sqrt{25-x^2}$ $[-4.5,3]$

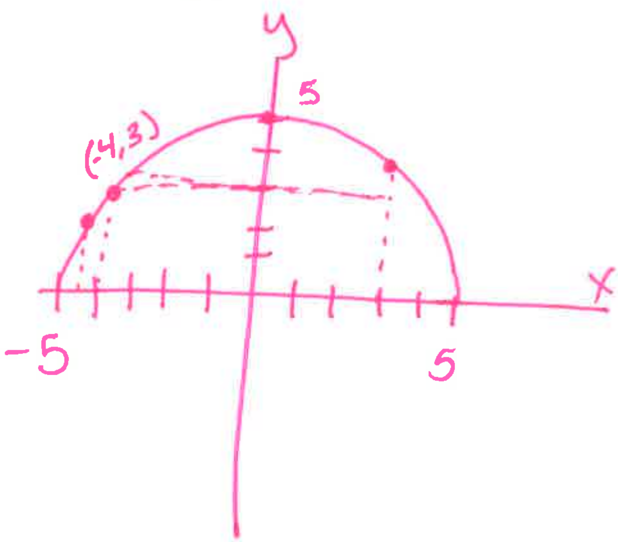
$f(c)=y=K=3$

$-5 \leq x \leq 5$ Good

$3^2=\sqrt{25-x^2}^{\,2}$

$9=25-x^2$
$-25 \quad -25$
$-16=-x^2$
$x^2=16$
$x=\pm\sqrt{16}$
$x=\pm 4$

$\boxed{x=-4 \text{ in bound}}$
$x=4$ not in bound



14) $\frac{1}{x-2}$ $[3,5]$

not continuous @ $x=2$ Asymptote

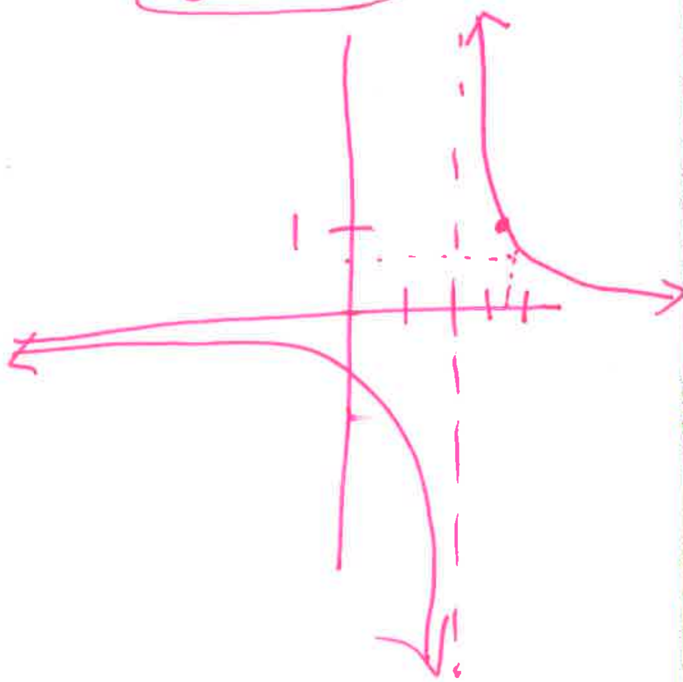
$f(3)=\frac{1}{1}=1$

$f(5)=\frac{1}{5-2}=\frac{1}{3}$

$\frac{1}{x-2}=\frac{5}{6}$

$5(x-2)=1(6)$
$5x-10=6$
$5x=16$
$x=16/5=\boxed{3\,1/5}$

$3<3\,1/5<5$

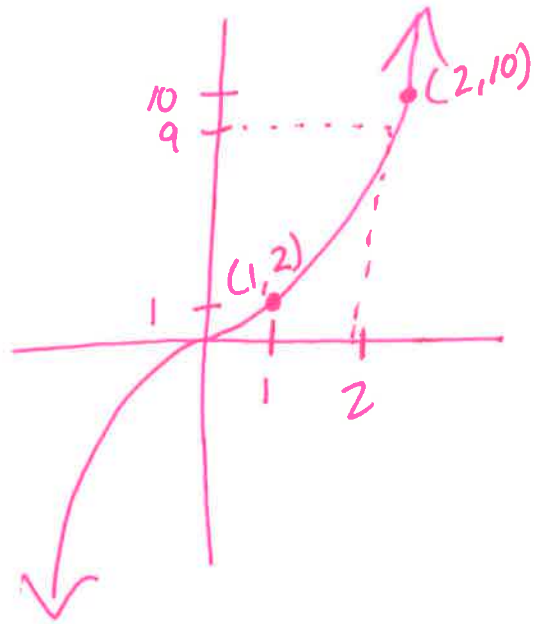


15) $f(x) = x^3 + x \quad [1, 2]$

$y = 9 \quad x = ?$

$9 = x^3 + x$

$x \approx 1.9$



16) $f(t) = \dfrac{t}{t+1} \quad [0, 1]$

$t = .499$ not continuous @ $t = -1$

By IVT it is continuous on $[0,1]$
So we must have a $t = .499$

17) $f(x) = \dfrac{x^2}{x^7+1}$ not continuous @ $x = -1$

So at $x = .4$ it is continuous and By IVT must have a Value.

$x^7 + 1 = 0$

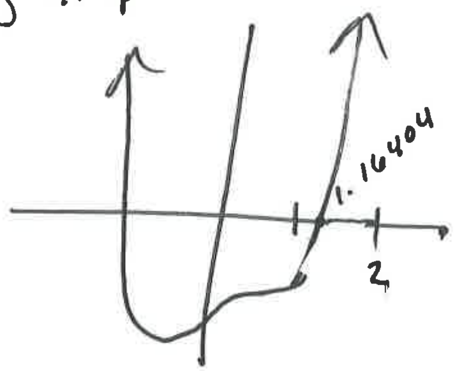
$x^7 = -1$

$\boxed{x = -1}$

18) $x^4 + x - 3 = 0 \quad [1,2]$

roots = zeros = x-intercept

By Graph:

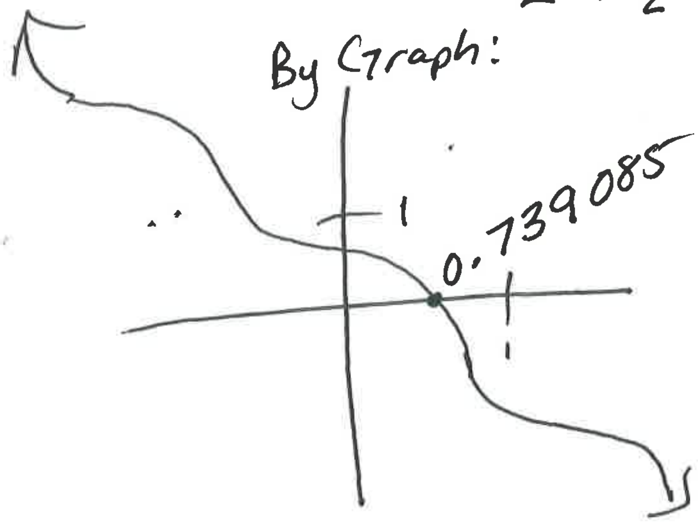


$\boxed{(1.16,0)}$

19) $\cos x - x = 0$

$[0, \frac{\pi}{2}]$

By Graph:



20) 0.32 first oz
0.23 per oz after

$f(x) = 0.23x + 0.32$

$1.00 = 0.23x + 0.32$

$0.68 = 0.23x$

$\boxed{x = 2.95652 \text{ oz}}$

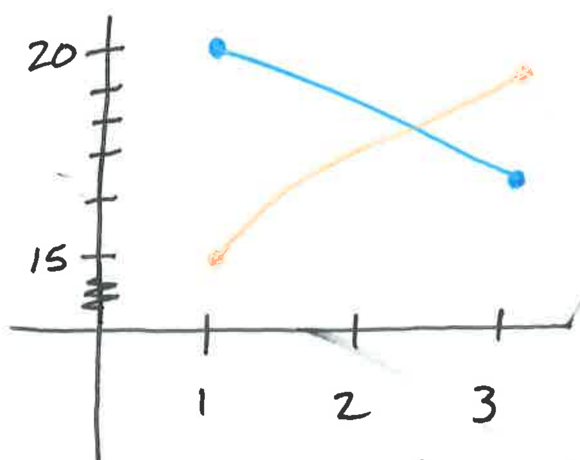
does it meet IVT?
yes it is continuous.

$y = \$1.00$

21)

1 min. in	
J	20 km/hr
K	15 km/hr

3 min in	
J	17 km/hr
K	19 km/hr



By IVT the most have a value for each y in between thus they must cross somewhere between 1 min and 3 min.