

Limit definition Derivatives and Tangent Lines

Use the limit definition of the derivative to (part A) find the slope of the tangent line and (part B) the equation of the tangent line. In part B your answer should be in slope-intercept form.

1) $y = x^3 - 3x^2 + 1$ at $x = -1$

$$\lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^3 - 3(x + \Delta x)^2 + 1 - (x^3 - 3x^2 + 1)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3 - 3(x^2 + 2x \Delta x + \Delta x^2) + 1 - x^3 + 3x^2 - 1}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3 - 3x^2 - 6x \Delta x - 3 \Delta x^2 + 1 - x^3 + 3x^2 - 1}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{3x^2 \cancel{\Delta x} + 3x \Delta x^2 + \Delta x^3 - 6x \Delta x - 3 \Delta x^2}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{3x^2 + 3x \Delta x + \Delta x^2 - 6x - 3 \Delta x^2}{\Delta x}$$

2) $y = 2x - 2$ at $x = 0$

$$A = 2$$

$$B = y = 2x - 2$$

$$y + 3 = 9(x + 1)$$

$$y + 3 = 9x + 9$$

$$y = 9x + 6$$

(B)

3) $y = -x^2 - 6x - 6$ at $(-3, 3)$

$$y' = -2x - 6$$

A [] B $y - 3 = 0(x + 3)$
y = 3

4) $y = \frac{2}{x-2}$ at $(1, -2)$

$$\lim_{\Delta x \rightarrow 0} \frac{\frac{2}{(x + \Delta x) - 2} - \frac{2}{x-2}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{2(x-2) - 2(x + \Delta x - 2)}{(x + \Delta x - 2)(x-2) \Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{2x - 4x - 2\Delta x + 4}{\Delta x(x + \Delta x - 2)(x-2)}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-2\Delta x}{\Delta x(x + \Delta x - 2)(x-2)} = \lim_{\Delta x \rightarrow 0} \frac{-2}{(x + \Delta x - 2)(x-2)}$$

6) $y = -6x - 9$ at $x = -1$

$$y' = -6$$

A = -6

B = y + 3 = -6(x + 1)

$$y + 3 = -6x - 6$$

$$y = -6x - 9$$

B = y + 2 = -2(x - 1)

$$y + 2 = -2x + 2$$

B = y = -2x

5) $y = -(2x - 2)^2$ at $x = -2$

$$\lim_{\Delta x \rightarrow 0} \frac{-(2(x + \Delta x) - 2)^2 - (-(2x - 2)^2)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-(2x + 2\Delta x - 2)^2 - (-(4x^2 - 8x + 4))}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-4x^2 - 8x\Delta x - 4\Delta x^2 + 8\Delta x + 8x + 4 - 4x^2 + 8x - 4}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-8x\Delta x - 4\Delta x^2 + 8\Delta x}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{-8x - 4\Delta x + 8}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-8x - 4\Delta x + 8}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{-8x + 8}{\Delta x} = \frac{-8(-2) + 8}{16 + 8} = \frac{A}{24}$$

7) $y = 2\tan(x)$ at $(0, 0)$

$$y' = \frac{2}{\cos^2 x}$$

A $m = \frac{2}{\cos^2(0)} = 2$

$$y - 0 = 2(x - 0)$$

B $y = 2x$

A = 0
B = -1

$$y + 36 = 24(x + 2)$$

$$y + 36 = 24x + 48$$

B $y = 24x + 12$

0) $\lim_{x \rightarrow 0} \tan(x) @ (0, 0)$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

8) $y = -\cos(x) @ (0, -1)$

$$= \lim_{\Delta x \rightarrow 0} -\frac{\cos(x + \Delta x) + \cos(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} -\frac{-\cos(x)\cos(\Delta x) + \sin(x)\sin(\Delta x) + \cos(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} -\left(\frac{\cos(x)\cos(\Delta x) - \cos(x) - \sin(x)\sin(\Delta x)}{\Delta x} \right)$$

$$= \lim_{\Delta x \rightarrow 0} -\left(\frac{\cos(x)\cos(\Delta x) - \cos(x)}{\Delta x} - \frac{\sin(x)\sin(\Delta x)}{\Delta x} \right)$$

$$= \lim_{\Delta x \rightarrow 0} -\left(\cos(x) \cdot \frac{\cos(\Delta x) - 1}{\Delta x} - \sin(x) \cdot \frac{\sin(\Delta x)}{\Delta x} \right)$$

$$= -\left(\lim_{\Delta x \rightarrow 0} + \cos(x) \cdot \lim_{\Delta x \rightarrow 0} \frac{\cos(\Delta x) - 1}{\Delta x} - \lim_{\Delta x \rightarrow 0} \sin(x) \cdot \lim_{\Delta x \rightarrow 0} \frac{\sin(\Delta x)}{\Delta x} \right)$$

$$= -\left(\cos(x) \cdot \lim_{\Delta x \rightarrow 0} \frac{\cos(\Delta x) - 1}{\Delta x} - \sin(x) \cdot \lim_{\Delta x \rightarrow 0} \frac{\sin(\Delta x)}{\Delta x} \right)$$

$$= -(\cos(x) \cdot 0 - \sin(x) \cdot (1))$$

$$= -(\overset{+}{0} - \sin(x))$$

$$= \sin(x)$$

a) $\sin(0) = 0$

b) $y = mx + b$

$$y = 0x - 1$$

$$y = -1$$

hint: $\tan(x + \Delta x) =$

7) $2 \tan(x) @ (0,0)$

$$\frac{\tan(x) + \tan(\Delta x)}{1 - \tan(x)\tan(\Delta x)}$$

a) $\lim_{\Delta x \rightarrow 0} \frac{2 \tan(x + \Delta x) - 2 \tan(x)}{\Delta x}$

$$= \lim_{\Delta x \rightarrow 0} \frac{2 \left(\frac{\tan(x) + \tan(\Delta x)}{1 - \tan(x)\tan(\Delta x)} \right) - 2 \tan(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{2 \left(\frac{\tan(x) + \tan(\Delta x)}{1 - \tan(x)\tan(\Delta x)} \right) - 2 \left(\frac{\tan x - \tan^2 x \tan(\Delta x)}{1 - \tan x \tan(\Delta x)} \right)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} 2 \left(\frac{(\tan(x) + \tan(\Delta x)) - (\tan(x) - \tan^2 x \tan(\Delta x))}{\Delta x (1 - \tan(x)\tan(\Delta x))} \right)$$

$$= \lim_{\Delta x \rightarrow 0} 2 \left(\frac{\tan(\Delta x) + \tan^2 x \tan(\Delta x)}{\Delta x (1 - \tan(x)\tan(\Delta x))} \right)$$

$$= \lim_{\Delta x \rightarrow 0} 2 \left(\frac{\tan(\Delta x)}{\Delta x} \cdot \frac{1 + \tan^2 x}{1 - \tan(x)\tan(\Delta x)} \right)$$

use $\tan \Delta x = \frac{\sin \Delta x}{\cos \Delta x}$ and $1 + \tan^2 x = \sec^2 x$

$$= \lim_{\Delta x \rightarrow 0} 2 \left(\frac{\sin(\Delta x)}{\Delta x} \cdot \frac{1}{\cos(\Delta x)} \right) \left(\frac{\sec^2 x}{1 - \tan(x)\tan(\Delta x)} \right)$$

$$= \lim_{\Delta x \rightarrow 0} 2 \cdot \lim_{\Delta x \rightarrow 0} \frac{\sin(\Delta x)}{\Delta x} \cdot \lim_{\Delta x \rightarrow 0} \frac{1}{\cos(\Delta x)} \cdot \lim_{\Delta x \rightarrow 0} \frac{\sec^2 x}{1 + \tan(x)\tan(\Delta x)}$$

by $\lim_{\Delta x \rightarrow 0} \frac{\sin(\Delta x)}{\Delta x} = 1$

$$= 2 \cdot 1 \cdot \frac{1}{1} \cdot \frac{\sec^2 x}{1} = 2 \sec^2 x = \frac{2}{\cos^2 x}$$

$$2) y = 2x - 2 \text{ @ } x=0$$

$$\lim_{\Delta x \rightarrow 0} \frac{2(x+\Delta x) - 2 - (2x - 2)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{2x + 2\Delta x - 2 - 2x + 2}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{2\Delta x}{\Delta x} = [2]$$

$$y+2 = 2(x-0)$$

$$\boxed{y+2 = 2x} \quad B$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$8) y = -\cos(x) \text{ @ } (0, -1)$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\cos(x+\Delta x) + \cos(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\cos(x)\cos(\Delta x) + \sin(x)\sin(\Delta x) + \cos(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\cos(\Delta x) + 1}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-\cos(\Delta x) + 1}{\Delta x}$$

$$= y' = \sin x$$

$$\textcircled{a) } \sin(0) = 0$$

$$y+1 = 0(x+0)$$

$$\boxed{y = -1}$$

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