

## Worksheet 1. What You Need to Know About Motion Along the $x$ -axis (Part 1)

In discussing motion, there are three closely related concepts that you need to keep straight. These are:

If  $x(t)$  represents the position of a particle along the  $x$ -axis at any time  $t$ , then the following statements are true.

1. "Initially" means when \_\_\_\_\_ = 0.
2. "At the origin" means \_\_\_\_\_ = 0.
3. "At rest" means \_\_\_\_\_ = 0.
4. If the velocity of the particle is positive, then the particle is moving to the \_\_\_\_\_.
5. If the velocity of the particle is \_\_\_\_\_, then the particle is moving to the left.
6. To find average velocity over a time interval, divide the change in \_\_\_\_\_ by the change in time.
7. Instantaneous velocity is the velocity at a single moment (instant!) in time.
8. If the acceleration of the particle is positive, then the \_\_\_\_\_ is increasing.
9. If the acceleration of the particle is \_\_\_\_\_, then the velocity is decreasing.
10. In order for a particle to change direction, the \_\_\_\_\_ must change signs.
11. One way to determine total distance traveled over a time interval is to find the sum of the absolute values of the differences in position between all resting points. Here's an example: If the position of a particle is given by:

$$x(t) = \frac{1}{3}t^3 - t^2 - 3t + 4,$$

find the total distance traveled on the interval  $0 \leq t \leq 6$ .

## Worksheet 2. Sample Practice Problems for the Topic of Motion (Part 1)

### Example 1 (numerical).

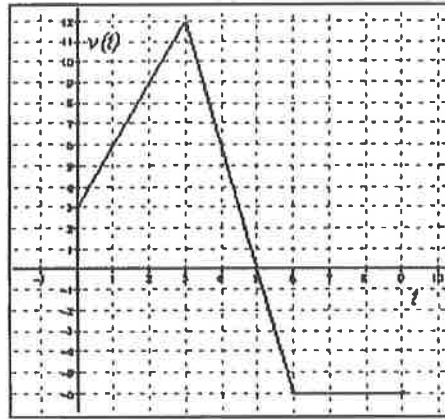
The data in the table below give selected values for the velocity, in meters/minute, of a particle moving along the  $x$ -axis. The velocity  $v$  is a differentiable function of time  $t$ .

Time $t$ (min)	0	2	5	6	8	12
Velocity $v(t)$ (meters/min)	-3	2	3	5	7	5

1. At  $t = 0$ , is the particle moving to the right or to the left? Explain your answer.
2. Is there a time during the time interval  $0 \leq t \leq 12$  minutes when the particle is at rest? Explain your answer.
3. Use data from the table to find an approximation for  $v'(10)$  and explain the meaning of  $v'(10)$  in terms of the motion of the particle. Show the computations that lead to your answer and indicate units of measure.
4. Let  $a(t)$  denote the acceleration of the particle at time  $t$ . Is there guaranteed to be a time  $t = c$  in the interval  $0 \leq t \leq 12$  such that  $a(c) = 0$ ? Justify your answer.

**Example 2 (graphical).**

The graph below represents the velocity  $v$ , in feet per second, of a particle moving along the  $x$ -axis over the time interval from  $t = 0$  to  $t = 9$  seconds.



1. At  $t = 4$  seconds, is the particle moving to the right or left? Explain your answer.
2. Over what time interval is the particle moving to the left? Explain your answer.
3. At  $t = 4$  seconds, is the acceleration of the particle positive or negative? Explain your answer.
4. What is the average acceleration of the particle over the interval  $2 \leq t \leq 4$ ? Show the computations that lead to your answer and indicate units of measure.
5. Is there guaranteed to be a time  $t$  in the interval  $2 \leq t \leq 4$  such that  $v'(t) = -3/2$  ft/sec<sup>2</sup>? Justify your answer.

6. At what time  $t$  in the given interval is the particle farthest to the right? Explain your answer.

**Example 3 (analytic).**

A particle moves along the  $x$ -axis so that at time  $t$  its position is given by:

$$x(t) = t^3 - 6t^2 + 9t + 11$$

1. At  $t = 0$ , is the particle moving to the right or to the left? Explain your answer.
2. At  $t = 1$ , is the velocity of the particle increasing or decreasing? Explain your answer.
3. Find all values of  $t$  for which the particle is moving to the left.
4. Find the total distance traveled by the particle over the time interval  $0 \leq t \leq 5$ .

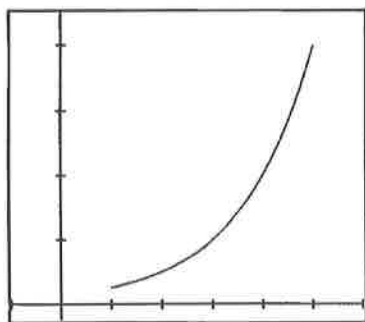
## Worksheet 3. Understanding the Relationships Among Velocity, Speed, and Acceleration

Speed is the absolute value of velocity. It tells you how fast something is moving without regard to the direction of movement.

1. What effect does absolute value have on numbers?
2. What effect does taking the absolute value of a function have on its graph?

For each situation below, the graph of a differentiable function giving velocity as a function of time  $t$  is shown for  $1 \leq t \leq 5$ , along with selected values of the velocity function. In the graph, each horizontal grid mark represents 1 unit of time and each vertical grid mark represents 4 units of velocity. For each situation, plot the speed graph on the same coordinate plane as the velocity graph and fill in the speed values in the table. Then, answer the questions below based on both the graph and the table of values.

### Situation 1: Velocity graph

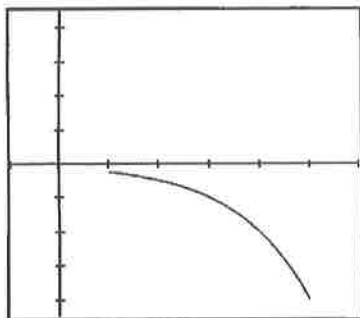


time	velocity	speed
1	1	
2	2	
3	4	
4	8	
5	16	

In this situation, the velocity is \_\_\_\_\_ and \_\_\_\_\_.  
 Positive or negative?      Increasing or decreasing?

Because velocity is \_\_\_\_\_, we know acceleration is \_\_\_\_\_.  
 Increasing or decreasing?      Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is \_\_\_\_\_.  
 Increasing or decreasing?

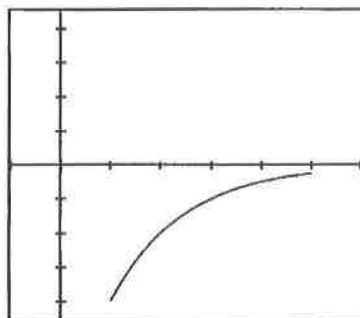
**Situation 2: Velocity graph**

time	velocity	speed
1	-1	
2	-2	
3	-4	
4	-8	
5	-16	

In this situation, the velocity is \_\_\_\_\_ and \_\_\_\_\_.  
 Positive or negative?      Increasing or decreasing?

Because velocity is \_\_\_\_\_, we know acceleration is \_\_\_\_\_.  
 Increasing or decreasing?      Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is \_\_\_\_\_.  
 Increasing or decreasing?

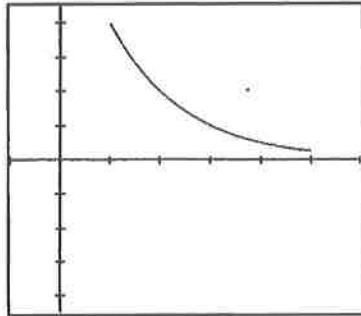
**Situation 3: Velocity graph**

time	velocity	speed
1	-16	
2	-8	
3	-4	
4	-2	
5	-1	

In this situation, the velocity is \_\_\_\_\_ and \_\_\_\_\_.  
 Positive or negative?      Increasing or decreasing?

Because velocity is \_\_\_\_\_, we know acceleration is \_\_\_\_\_.  
 Increasing or decreasing?      Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is \_\_\_\_\_.  
 Increasing or decreasing?

**Situation 4: Velocity graph**

time	velocity	speed
1	16	
2	8	
3	4	
4	2	
5	1	

In this situation, the velocity is \_\_\_\_\_ and \_\_\_\_\_.  
 Positive or negative?      Increasing or decreasing?

Because velocity is \_\_\_\_\_, we know acceleration is \_\_\_\_\_.  
 Increasing or decreasing?      Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is \_\_\_\_\_.  
 Increasing or decreasing?

**Conclusion:**

In which situations was the speed increasing? \_\_\_\_\_

When the speed is increasing, the velocity and acceleration have \_\_\_\_\_ signs.  
 Same or opposite?

In which situations was the speed decreasing? \_\_\_\_\_

When the speed is decreasing, the velocity and acceleration have \_\_\_\_\_ signs.  
 Same or opposite?

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**Assessing Students' Understanding (A Short Quiz):**

1. If velocity is negative and acceleration is positive, then speed is \_\_\_\_\_.
2. If velocity is positive and speed is decreasing, then acceleration is \_\_\_\_\_.
3. If velocity is positive and decreasing, then speed is \_\_\_\_\_.
4. If speed is increasing and acceleration is negative, then velocity is \_\_\_\_\_.
5. If velocity is negative and increasing, then speed is \_\_\_\_\_.
6. If the particle is moving to the left and speed is decreasing, then acceleration is \_\_\_\_\_.