Position/Velocity/Acceleration

Practice Problems:

1. Bugs Bunny has been captured by Yosemite Sam and forced to walk the plank. Instead of waiting for Yosemite Sam to finish cutting the board from underneath him, Bugs finally decides just to jump. Bugs' position, s, is given by the equation $s = \frac{1}{2}gt^2 + v_0t + s_0$. He jumps up at 16 ft/sec from a plank 320 feet high.
   a. What is the position equation and velocity equation?
   b. When will Bugs hit the ground?
   c. What is Bugs' velocity at impact?
   d. What is Bugs' speed at impact?
   e. When does Bugs reach a maximum height?
   f. What is Bugs' maximum height?
   g. Find Bugs' velocity at 1 second.
   h. Find Bugs' average velocity from the time he jumped until the time he landed.

2. Spider-Man goes to the top of a building 320 feet high to prevent Dr. Octopus from destroying the Earth. He knocks Dr. Octopus unconscious and throws him up in the air at 128 ft/sec.

   a. Find the position, velocity, and acceleration as functions of time.
   b. How long will it take for Dr. Octopus to reach his maximum height?
   c. What is the maximum height that Dr. Octopus reaches?
   d. When does Dr. Octopus hit the ground?
   e. With what velocity does Dr. Octopus hit the ground?

3. A student throws his Calculus book straight up from the ground at 490 m/sec.

   a. Find the position, velocity, and acceleration as functions of time.
   b. How long will it take the book to reach its maximum height?
   c. What is the maximum height the book goes?
   d. When does the book hit the ground?
   e. With what velocity does the book hit the ground?

4. A student drops his graphing calculator from the top level of the parking garage 64 feet high.

   a. Find the position, velocity, and acceleration as functions of time.
   b. When does the calculator hit the ground?
   c. With what velocity does the calculator hit the ground?
   d. What was the calculator's average velocity from the time it was dropped until it hit the ground?
   e. Was this smart to drop such an expensive calculator from so high up?

Key

1)

ft/sec

$S = \frac{1}{2}gt^2 + V_0 t + S_0$ $\quad V_0 = 16$ ft/sec $\quad S_0 = 320$ ft

$g = -32$ ft/sec^2

a) $S(t) = \frac{1}{2}(32)t^2 + 16t + 320$

$S(t) = -16t^2 + 16t + 320$

$V(t) = -32t + 16$

b) $S(t) = 0$

$-16t^2 + 16t + 320 = 0$

$t = \frac{-16 \pm \sqrt{16^2 - 4(-16)(320)}}{2(-16)} = \frac{-16 \pm \sqrt{256 + 20480}}{-32}$

~~$t = -4$~~ $\boxed{t = 5}$

hits the ground @ $t = 5$ seconds

c) @ $t = 5$

$V(5) = -32(5) + 16$

$= -160 + 16$

$= -144$ ft/sec.

d) speed = |Velocity|

speed = $|-144|$

speed = 144 ft/sec

e) Max height is where direction changes.

@ $V(t) = 0$

$V(t) = -32t + 16 = 0$

$-32t = -16$

$t = \frac{1}{2}$ sec

f) $S(\frac{1}{2}) = -16(\frac{1}{2})^2 + 16(\frac{1}{2}) + 320 = 324$ ft

g) $V(1) = -32(1) + 16 = -16$ ft/sec @ $t = 1$

h) $t = 0$ (start) to $t = 5$ (land) $\quad$ Avg V $= \frac{S(5) - S(0)}{5 - 0}$

$= \frac{-16(5)^2 + 16(5) + 320 - (0 + 0 + 320)}{5} = -64$ ft/

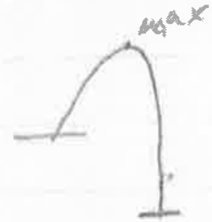
ft/sec

2)

$$\tfrac{1}{2}(g)$$
$$S(t) = \tfrac{1}{2}(-32)t^2 + V_0 t + S_0$$
$$= -16t^2 + 128t + 320$$

$S_0 = 320$ ft
$V_0 = 128$ ft/sec

a) $S(t) = -16t^2 + 128t + 320$
$V(t) = S'(t) = -16(2)t + 128 + 0$
$V(t) = -32t + 128$
$A(t) = V'(t) = S''(t) = -32$



b) time @ max height $V(t) = 0$ Velocity changes direction
$-32t + 128 = 0$
$-32t = -128$
$t = 4$ sec.

c) max height @ $t = 4$
$S(4) = -16(4)^2 + 128(4) + 320$
$S(4) = 576$ ft is the max height

d) $S(t) = 0$
$-16t^2 + 128t + 320 = 0$
$t \neq -2$ and $\boxed{t = 10 \text{ sec.}}$

e) $V(10) = -32(10) + 128$
$= -192$ ft/sec.

3)

m/sec gravity = $-9.8\ m/sec^2$

$V_0 = 490$ $S_0 = 0$ $S(t) = \frac{1}{2}(-9.8)t^2 + V_0 t + S_0$

a) $S(t) = -4.9t^2 + 490t + 0$

$V(t) = -9.8t + 490$

$A(t) = -9.8$

b) time @ max height $V(t) = 0$

$-9.8t + 490 = 0$

$-9.8t = -490$

$t = 50$ seconds

c) max height

$S(50) = -4.9(50)^2 + 490(50)$

$= 12250$ meters

d) $S(t) = 0$ hits the ground

$-4.9t^2 + 490t = 0$

$t \neq 0$ and $t = 100$ seconds

e) $V(100) = -9.8(100) + 490$

$= -490$ m/sec

4)

$S_0 = 64$ ft free fall $V_0 = 0$ $g = -32$ ft/sec

a) $S(t) = \frac{1}{2} g t^2 + V_0 t + S_0$

$= \frac{1}{2}(-32)t^2 + 0t + 64$

$= -16t^2 + 64$

$V(t) = -32t$

$A(t) = -32$

b) $S(t) = 0$

$-16t^2 + 64 = 0$

$-16t^2 = -64$

$t^2 = 4$

$t = \pm\sqrt{4}$

$t = 2, \cancel{-2}$

c) $V(2) = -32(2)$

$= -64$ ft/sec

d) avg $V = \frac{S(b) - S(a)}{b - a}$ $[0, 2]$

$= \frac{(-16(2)^2 + 64) - (-16(0)^2 + 64)}{2 - 0}$

$= -32$ ft/sec

e) Never