Related Rates

1. An oil spill is growing in a circular shape. The radius is increasing at the rate of 5 miles per day. How fast is the area changing when the circumference is 49π miles?

2. A 17 foot ladder is leaning against a wall. It is sliding down the wall at a rate of 2ft/min.
   a) How fast is the ladder moving away from the wall when the top is 15 feet from the ground?
   b) How fast is the area enclosed by the ladder changing when the top is 15 feet from the ground ? elevation
   c) How fast is the angle changing when the top is 15 feet from the ground?

3. A cone with diameter 14 and height 28 is being filled with oil at a rate of 2 cubic feet per second.
   a) Express the volume of the water as a function of the water level h.
   b) How fast is the level of the oil rising in the cone when h = 4?
   c) How fast is the radius increasing when h = 4?

4. A man observes the launching of a rocket from a distance of 300 feet. The rocket is launched at a speed of 100 ft/sec. How fast is the rocket moving away from the man four seconds after launch? How fast is the angle changing at this same moment?

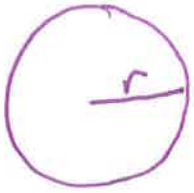
5. A spherical balloon is being inflated at the rate of $2in^3$/sec. How fast is the radius increasing when the radius is 10 inches?

6. A six foot tall man is walking towards a light pole that is ten feet tall at a speed of two feet per second. How fast is the shadow moving when he is eight from the lamp?

7. Melted chocolate is flowing at a rate of $3m^3/s$ into a giant ice cream cone with diameter 30m and height 180m.
   a) Find the volume of the melted chocolate as a function of the chocolate level h.
   b How fast is the level of the cone rising when the height is 3m?
   c How fast is the radius changing when the height is 3m?

Related Rates Key

1)



$\frac{dr}{dt} = 5\frac{m}{d}$ $\quad \frac{dA}{dt} = ?$ $\quad C = 49\pi$ miles

$C = d\pi$

$C = 2r\pi$

$C = 24.5(2)\pi$

$r = 24.5$

$A = \pi r^2$

$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2)$

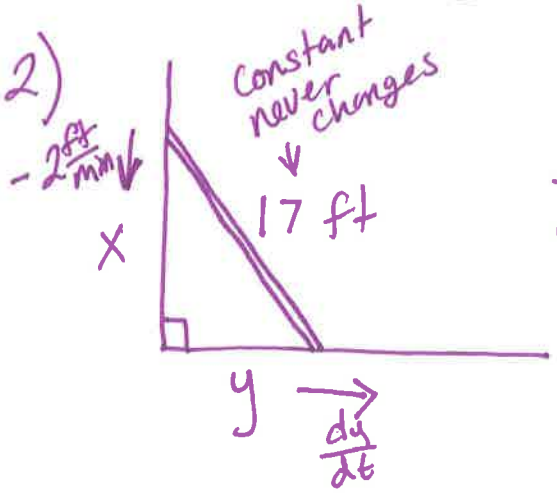
$\frac{dA}{dt}(1) = \pi 2r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(24.5)(5)$

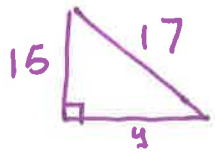
$\boxed{\frac{dA}{dt} = 245\pi \frac{m^2}{day}}$

2)



$\frac{dx}{dt} = -2\frac{ft}{min}$ $\quad x^2 + y^2 = 17^2$

a) $\frac{dy}{dt} = ?$ $\quad x = 15$



$15^2 + y^2 = 17^2$

$y = \pm\sqrt{17^2 - 15^2}$

$y = \sqrt{64} = 8$

$\frac{d}{dt}x^2 + \frac{d}{dt}y^2 = \frac{d}{dt}17^2$

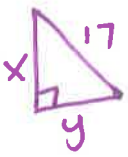
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(15)(-2) + 2(8)\frac{dy}{dt} = 0$

$-60 + 16\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{60}{16} = \boxed{\frac{15}{4}\frac{ft}{min}}$

2) b) $\frac{dA}{dt} = ?$ $x = 15$ ft



$A = \frac{1}{2} b(h)$; $b = y$, $h = x$

$A = \frac{1}{2} y(x)$

from problem $= \frac{dx}{dt} = -2$

from part a $= \frac{dy}{dt} = \frac{15}{4}$

$\frac{d}{dt} A = \frac{d}{dt}\left(\frac{1}{2} yx\right)$

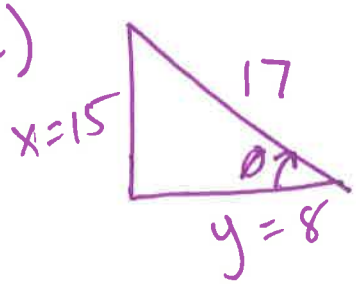
$\frac{dA}{dt} = \frac{1}{2}\left((1)x\frac{dy}{dt} + (1)y\frac{dx}{dt}\right)$

$\frac{dA}{dt} = \frac{1}{2}\left((15)\left(\frac{15}{4}\right) + (8)(-2)\right)$

$\frac{dA}{dt} = 28.125 - 8$

$\frac{dA}{dt} = 20.125 \frac{ft^2}{min}$

c)



$\frac{d\theta}{dt} = ?$

$\frac{dy}{dt} = \frac{15}{4}\frac{dx}{dt} = -2$

$\sin(\theta) = \frac{x}{17}$

$\frac{d\theta}{dt}\cos(\theta) = \frac{1}{17}\frac{dx}{dt}$

$\frac{d\theta}{dt}\left(\frac{8}{17}\right) = \frac{1}{17}(-2)$

$\frac{d\theta}{dt} = \frac{-2}{17}\left(\frac{17}{8}\right)$

$\frac{d\theta}{dt} = -\frac{1}{4} \frac{rad}{min}$

3)

$d = 14$

$r = 7$

$\frac{dV}{dt} = 2\frac{ft^3}{sec}$

$V = \frac{1}{3}\pi r^2 h$

$4r = h$

$2r = d = h$

~~$r^2 = h^2$~~

$4r = h$

$r = \frac{h}{4}$

a) $V = \frac{1}{3}\pi \left(\frac{h}{4}\right)^2 h$

$\boxed{V = \frac{1}{3}\pi \frac{h^3}{16}}$

$V(h) =$

b) $\frac{dh}{dt} = ?$ $h = 4$ $V = \frac{1}{3}\pi \frac{h^3}{16}$

$\frac{dV}{dt} = \frac{3}{3}\pi h \frac{21}{16}\frac{dh}{dt}$

$2 = \pi \frac{(4)^2}{16}\frac{dh}{dt}$

$\frac{2}{\pi(4)^2} = \frac{dh}{dt}$

$\frac{2}{\pi 8} = \boxed{\frac{dh}{dt} = \frac{2}{8\pi}\ ft/sec}$

c) $\frac{dr}{dt} = ?$ $h = 4$ $h = 4$ $r = 1$

from part b. $= \frac{dh}{dt} = \frac{2}{8\pi}$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + r^2(1)\frac{dh}{dt}\right)$

$2 = \frac{1}{3}\pi\left(2(1)\frac{dr}{dt}(4) + (1^2)\left(\frac{2}{8\pi}\right)\right)$

$2 = \frac{1}{3}\pi\left(8\frac{dr}{dt} + \frac{2}{8\pi}\right)$

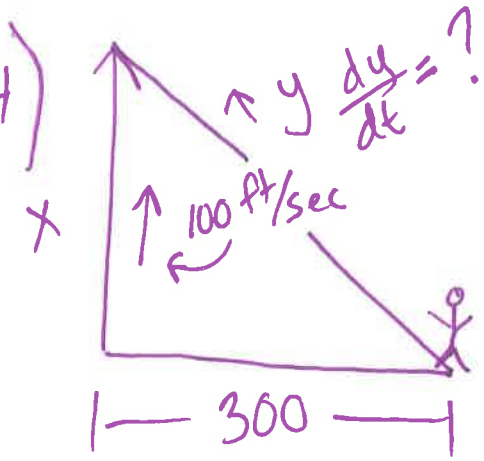
$\frac{6}{\pi} = 2\frac{dr}{dt} + \frac{32}{\pi}$

$\frac{6}{\pi} - \frac{2}{\pi} = 8\frac{dr}{dt}$

$\frac{4}{\pi} = 8\frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{1}{2\pi}}$

4)

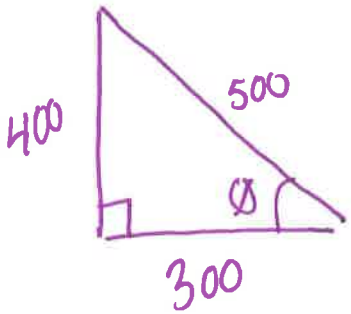


Speed = 100 ft/sec

$t = 4$ sec

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

@ $t = 4$



a) $x^2 + 300^2 = y^2$

$\frac{dy}{dt} = ?$

$$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$$

$$2(400)(100) + 0 = 2(500)\frac{dy}{dt}$$

$$80000 + 0 = 1000\frac{dy}{dt}$$

$$\boxed{\frac{dy}{dt} = 80.\ \text{ft/sec}}$$

b) $\cos\theta = \frac{300}{y}$

$$-\sin\theta\frac{d\theta}{dt} = -300y^{-2}\frac{dy}{dt}$$

$$-\left(\frac{400}{500}\right)\frac{d\theta}{dt} = \frac{-300}{(500)^2}(80.)$$

$$\frac{d\theta}{dt} = -0.096 \quad \left(-\frac{5}{4}\right)$$

$$\boxed{\frac{d\theta}{dt} = 0.12 \ \frac{\text{rad}}{\text{sec}}}$$

5)

$\frac{dV}{dt} = 2 \text{ in}^3/\text{sec}$ $\quad \frac{dr}{dt} = ?$

$r = 10$ in

$V = \frac{4}{3}\pi r^3$

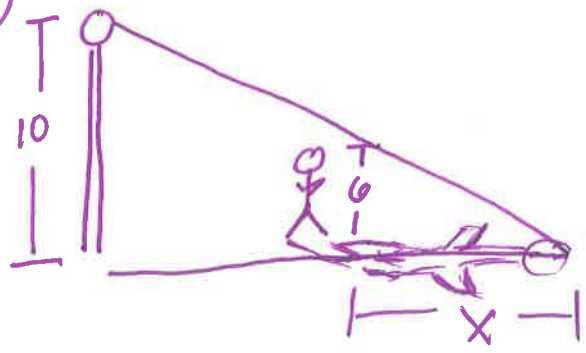
$\frac{dV}{dt} = \frac{4}{3}\pi (3r^2)\frac{dr}{dt}$

$2 = \frac{4}{3}\pi (10)^2 \left(\frac{dr}{dt}\right)$

$2 = 400\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{2}{400\pi} = \boxed{\frac{1}{200\pi} \text{ in/sec}}$

6)



$y = 8$

$\frac{dy}{dt} = -2$ $\quad \frac{dx}{dt} = ?$

Speed $= \frac{\text{distance}}{\text{time}}$

$\frac{10}{y+x} = \frac{6}{x}$

$10x = 6(y+x)$

$10\frac{dx}{dt} = 6\left(1\frac{dy}{dt} + 1\frac{dx}{dt}\right)$

$10\frac{dx}{dt} = 6\left(1(-2) + 1\frac{dx}{dt}\right)$

$10\frac{dx}{dt} = -12 + 6\frac{dx}{dt}$

$10\frac{dx}{dt} - 6\frac{dx}{dt} = -12$

$4\frac{dx}{dt} = -12$

$\frac{dx}{dt} = -\frac{12}{4}$

$\boxed{\frac{dx}{dt} = -3 \text{ ft/sec}}$

7)

$D = 30m$

$h = 180m$

$d = 2r$

$\frac{dV}{dt} = 3\ m^3/s$

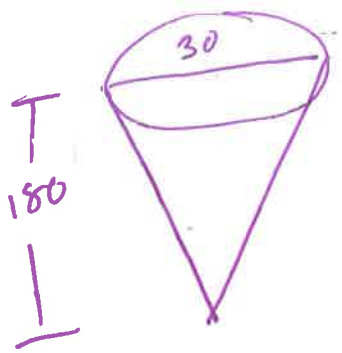
$180h = 30d$

$h = \frac{3}{8}d$

$h = \frac{3}{8}(2r)$

$h = \frac{3}{4}r$

$\frac{4}{3}h = r$



$\frac{30}{180} = \frac{r}{3}$

$90 = 180r$

$r = \frac{90}{180} = \frac{1}{2}$

a) $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h$

$V = \frac{1}{3}\pi\frac{16}{9}h^3 = \frac{16\pi}{27}h^3$

b) $\frac{dh}{dt} = ?\quad h = 3$

$\frac{dV}{dt} = \frac{16}{9}\pi h^2\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{16}{9}\pi h^2\frac{dh}{dt}$

$3 = \frac{16}{9}\pi(3)^2\frac{dh}{dt}$

$\frac{3}{16\pi} = \frac{dh}{dt} = \frac{3}{16\pi}\ m/s$

c) $\frac{dr}{dt} = ?\quad h = 3$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + (1)r^2\frac{dh}{dt}\right)$

$\left(\frac{3}{1\pi}\right)\ 3 = \frac{1}{3}\pi\left(2\left(\frac{1}{2}\right)\frac{dr}{dt}(3) + 3^2\left(\frac{3}{16\pi}\right)\right)$

$\frac{9}{\pi} = 3\frac{dr}{dt} + \frac{27}{16\pi}$

$\frac{117}{16\pi} = 3\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{117}{3(16\pi)}$

$\boxed{\frac{dr}{dt} = \frac{39}{16\pi}}$