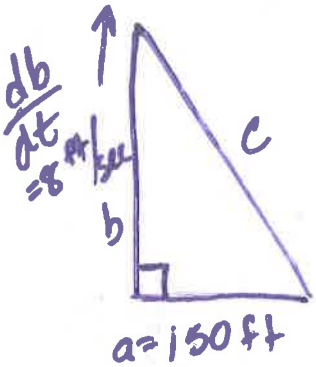
AP Calculus
Related Rates Worksheet

1. A small balloon is released at a point 150 feet from an observer, who is on level ground. If the balloon goes straight up at a rate of 8 feet per second, how fast is the distance from the observer to the balloon increasing when the balloon is 50 feet high?



$\frac{dc}{dt} = ?$

$b = 50\text{ ft}$

$a^2 + b^2 = c^2$

$150^2 + b^2 = c^2$

$0 + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

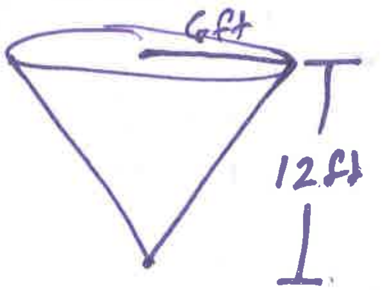
$2(50)8 = 2(50\sqrt{10})\frac{dc}{dt}$

$150^2 + 50^2 = c^2$

$c = 50\sqrt{10}$

$\frac{dc}{dt} = \frac{800}{100\sqrt{10}} = 2.5298$ ft/sec

2. Water is pouring into a conical tank at the rate of 8 cubic feet per minute. If the height of the tank is 12 feet and the radius of its circular opening is 6 feet, how fast is the water level rising when the water is 4 feet deep?



$\frac{dV}{dt} = 8\ \text{ft}^3/\text{min}$

$\frac{dh}{dt} = ?$

$h = 4$

$h = 2r \quad r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{3}\pi\, h^3/4$

$V = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{\cancel{12}_{4}}\cancel{3}h^2\frac{dh}{dt}$

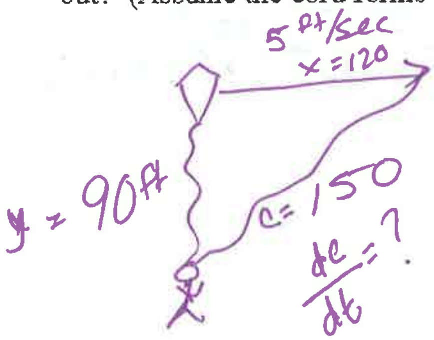
$8 = \frac{\pi}{4}(4)^2\frac{dh}{dt}$

$8 = \frac{16\pi}{4}\frac{dh}{dt}$

$\frac{8 \cdot 4}{16\pi} = \frac{dh}{dt}$

$= \frac{2}{\pi}$ ft/min

3. A child is flying a kite. If the kite is 90 feet above the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child letting out the cord when 150 feet of cord is out? (Assume the cord forms a straight line – i.e. there is no slack in the string.)



$x^2 + y^2 = c^2$

$x^2 + 90^2 = c^2$

$2x\frac{dx}{dt} + 0 = 2c\frac{dc}{dt}$

$2(120)(5) = 2(150)\frac{dc}{dt}$

$\frac{1200}{300} = \frac{300\frac{dc}{dt}}{300}$

$\frac{dc}{dt} = 4$ ft/sec.

AP Calculus
Related Rates/Implicit Differentiation Worksheet

1. Sand is falling off a conveyor belt at a rate of 12 cubic feet per minute into a conical pile. The diameter of the pile is four times the height. At what rate is the height of the pile changing when the pile is 10 feet high?

$\frac{dV}{dt} = 12 \frac{ft^3}{min}$ $\quad \frac{dh}{dt} = ?$ $\quad h = 10$

$d = 4h$

$2r = 4h$

$r = 2h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (2h)^2 h$

$V = \frac{1}{3}\pi 4 h^3$

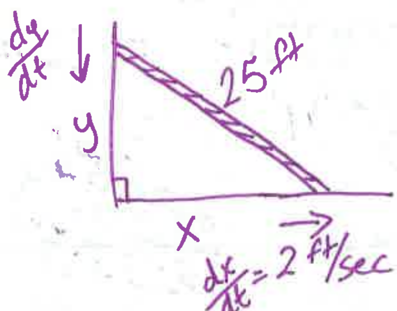
$\frac{dV}{dt} = \frac{\cancel{3}}{\cancel{3}}\pi 4 h^2 \frac{dh}{dt}$

$12 = \pi 4 (10)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{3\pi}{100} = 0.03\pi \frac{ft}{min}$

2. A ladder 25 feet long is leaning against a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second.

a. How fast is the top of the ladder moving down the wall when the base of the ladder is 12 feet from the wall?



$x^2 + y^2 = 25^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(12)(2) + 2(\sqrt{481})\frac{dy}{dt} = 0$

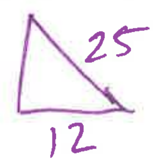
$2\sqrt{481}\frac{dy}{dt} = -48$

$\frac{dy}{dt} = -1.0943$ ft/sec

$\frac{dy}{dt} = -?$

$x = 12$

$y = \sqrt{481}$



b. Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base is 12 feet from the wall.

$\frac{dA}{dt} = ?$ $\quad x = 12$

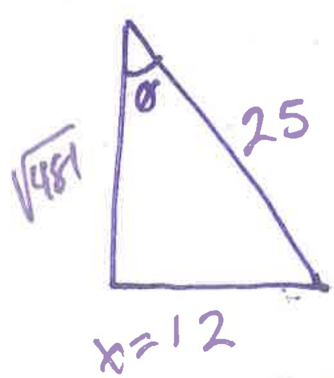
$A = \frac{1}{2} b \cdot h$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{db}{dt} h + \frac{dh}{dt} b\right)$

$= \frac{1}{2}\left(2(\sqrt{481}) + (-1.0943)12\right) =$

$\frac{dA}{dt} = 15.3669\ ft^2/sec$

c. Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 12 feet from the wall.



$\sin(\theta) = \frac{x}{25}$

$\cos(\theta)\frac{d\theta}{dt} = \frac{1}{25}\frac{dx}{dt}$

$\frac{\sqrt{481}}{25}\frac{d\theta}{dt} = \frac{1}{25}(2)$

$\frac{d\theta}{dt} = \frac{2}{\cancel{25}} \cdot \frac{\cancel{25}}{\sqrt{481}} = \frac{2}{\sqrt{481}} = 0.9119$ rad/sec

Differentiate implicitly.

3. $4x^2 + 3y^2 - 3x = 20$

$4\cdot 2x\frac{dx}{dx} + 3\cdot 2y\frac{dy}{dx} - 3\frac{dx}{dx} = 0$

$8x + 6y\frac{dy}{dx} - 3 = 0$

$6y\frac{dy}{dx} = -8x + 3$

$\frac{dy}{dx} = \frac{-8x+3}{6y}$

4. $\cos x + \sin 3y = 3$

$-\sin x\frac{\cancel{dy}}{dx} + \cos(3y)3\frac{dy}{dx} = 0$

$3\cos(3y)\frac{dy}{dx} = \sin(x)$

$\frac{dy}{dx} = \frac{\sin(x)}{3\cos(3y)}$

5. Determine y'' for the equation: $x^2 + 2y^2 - x = 5$

$2x\frac{dx}{dx} + 4y\frac{dy}{dx} - 1\frac{dx}{dx} = 0$

$2x + 4y\frac{dy}{dx} - 1 = 0$

$4y\frac{dy}{dx} = -2x + 1$

$\frac{dy}{dx} = \frac{-2x+1}{4y}$

$\frac{d^2y}{dx^2} = \frac{(-2\frac{dx}{dx}+0)(4y) - (4\frac{dy}{dx})(-2x+1)}{(4y)^2}$

$= \frac{-2(4y) - 4\frac{dy}{dx}(-2x+1)}{16y^2}$

$= \frac{-8y - 4(\frac{-2x+1}{4y})(-2x+1)}{16y^2}$

$= \frac{-8y^2 - (1-2x)^2}{16y^3}$

6. Find equations of the tangent and normal lines to $f(x) = \sqrt[3]{1-2x} + x^3$ at x = 1. Give your answer in standard form (Ax + By + C = 0).

$f(x) = (1-2x)^{1/3} + x^3$

$f'(1) = \frac{1}{3}(1-2x)^{-2/3}\cdot(-2) + 3x^2$

$= \frac{1}{3}(1-2(1))^{-2/3}\cdot(-2) + 3(1)^2$

$m = 7/3 \quad (1,0)$

Tangent: $y - 0 = 7/3(x-1)$

$y = 7/3x - 7/3$

$3y = 7x - 7$

$-7x + 3y + 7 = 0$

normal: $m = -3/7 \quad (1,0)$

$y - 0 = -3/7(x-1)$

$y = -3/7x + 3/7$

$7y = -3x + 3$

$3x + 7y - 3 = 0$