

## Chain Rule Day 2

Differentiate each function with respect to  $x$ .

$$1) y = \left( (4x^5 - 1)^{\frac{1}{4}} + 3 \right)^{\frac{1}{5}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{5} \left( (4x^5 - 1)^{\frac{1}{4}} + 3 \right)^{-\frac{4}{5}} \cdot \frac{1}{4} (4x^5 - 1)^{-\frac{3}{4}} \cdot 20x^4 \\ &= \frac{x^4}{\left( (4x^5 - 1)^{\frac{1}{4}} + 3 \right)^{\frac{4}{5}} \cdot (4x^5 - 1)^{\frac{3}{4}}} \end{aligned}$$

$$2) y = \left( (3x^3 - 4)^{-5} - 1 \right)^{\frac{1}{4}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{4} \left( (3x^3 - 4)^{-5} - 1 \right)^{-\frac{3}{4}} \cdot -5(3x^3 - 4)^{-6} \cdot 9x^2 \\ &= \frac{45x^2}{4 \left( (3x^3 - 4)^5 + 1 \right)^{\frac{3}{4}} \cdot (3x^3 - 4)^9} \end{aligned}$$

$$3) y = \left( (-5x^3 + 2)^{-2} - 4 \right)^{-3}$$

$$\begin{aligned} \frac{dy}{dx} &= -3 \left( (-5x^3 + 2)^{-2} - 4 \right)^{-4} \cdot -2(-5x^3 + 2)^{-3} \cdot -15x^2 \\ &= -\frac{90x^2(-5x^3 + 2)^5}{(-4(-5x^3 + 2)^2 + 1)^4} \end{aligned}$$

$$4) y = \left( (-2x + 1)^{\frac{1}{3}} + 4 \right)^{\frac{1}{2}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2} \left( (-2x + 1)^{\frac{1}{3}} + 4 \right)^{-\frac{1}{2}} \cdot \frac{1}{3} (-2x + 1)^{-\frac{2}{3}} \cdot -2 \\ &= -\frac{1}{3 \left( (-2x + 1)^{\frac{1}{3}} + 4 \right)^{\frac{1}{2}} \cdot (-2x + 1)^{\frac{2}{3}}} \end{aligned}$$

$$5) y = \sqrt[3]{-x^2 + 3} \cdot \sqrt[5]{5x + 3}$$

$$\begin{aligned} \frac{dy}{dx} &= (-x^2 + 3)^{\frac{1}{3}} \cdot \frac{1}{5} (5x + 3)^{-\frac{4}{5}} \cdot 5 + (5x + 3)^{\frac{1}{5}} \cdot \frac{1}{3} (-x^2 + 3)^{-\frac{2}{3}} \cdot -2x \\ &= \frac{-13x^2 + 9 - 6x}{3(5x + 3)^{\frac{4}{5}} \cdot (-x^2 + 3)^{\frac{2}{3}}} \end{aligned}$$

$$6) y = (4x^5 + 1)^{-2} \sqrt[4]{2x - 1}$$

$$\begin{aligned} \frac{dy}{dx} &= (4x^5 + 1)^{-2} \cdot \frac{1}{4} (2x - 1)^{-\frac{3}{4}} \cdot 2 + (2x - 1)^{\frac{1}{4}} \cdot -2(4x^5 + 1)^{-3} \cdot 20x^4 \\ &= \frac{-156x^5 + 1 + 80x^4}{2(2x - 1)^{\frac{3}{4}} \cdot (4x^5 + 1)^3} \end{aligned}$$

$$7) y = (2x + 3)^{-3} \cdot (-2x^4 + 1)^{-2}$$

$$\begin{aligned} \frac{dy}{dx} &= (2x + 3)^{-3} \cdot -2(-2x^4 + 1)^{-3} \cdot -8x^3 + (-2x^4 + 1)^{-2} \cdot -3(2x + 3)^{-4} \cdot 2 \\ &= \frac{2(22x^4 + 24x^3 - 3)}{(2x + 3)^4 \cdot (-2x^4 + 1)^3} \end{aligned}$$

$$8) y = \frac{(-x^5 + 3)^{-2}}{(3x + 2)^{-4}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(3x + 2)^{-4} \cdot -2(-x^5 + 3)^{-3} \cdot -5x^4 - (-x^5 + 3)^{-2} \cdot -4(3x + 2)^{-5} \cdot 3}{((3x + 2)^{-4})^2} \\ &= \frac{2(3x + 2)^3(9x^5 + 10x^4 + 18)}{(-x^5 + 3)^3} \end{aligned}$$