Calculus

Name_____

Differentiation Rules Review

Date_____

Differentiate each function with respect to x.

1) $y = 2x$

$$\frac{dy}{dx} = 2$$

2) $y = \frac{1}{2}x^{-1} + \frac{5}{2}x^{-3}$

$$\frac{dy}{dx} = -\frac{1}{2x^2} - \frac{15}{2x^4}$$

3) $y = 3x^{\frac{2}{3}} + x^{-1} - \frac{5}{3}x^{-2}$

$$\frac{dy}{dx} = \frac{2}{x^{\frac{1}{3}}} - \frac{1}{x^2} + \frac{10}{3x^3}$$

4) $y = 2x^5(-4x^4 + 1)$

$$\frac{dy}{dx} = 2x^5 \cdot -16x^3 + (-4x^4 + 1) \cdot 10x^4$$
$$= -72x^8 + 10x^4$$

5) $y = (3x^3 - 5)(5x^4 - 3)$

$$\frac{dy}{dx} = (3x^3 - 5) \cdot 20x^3 + (5x^4 - 3) \cdot 9x^2$$
$$= 105x^6 - 100x^3 - 27x^2$$

6) $y = (\sqrt[4]{x} + 2)(2x^2 + 1)$

$$\frac{dy}{dx} = \left(x^{\frac{1}{4}} + 2\right) \cdot 4x + (2x^2 + 1) \cdot \frac{1}{4}x^{-\frac{3}{4}}$$
$$= \frac{9x^{\frac{5}{4}}}{2} + 8x + \frac{1}{4x^{\frac{3}{4}}}$$

7) $y = \frac{2x^2}{3x^5 + 3}$

$$\frac{dy}{dx} = \frac{(3x^5 + 3) \cdot 4x - 2x^2 \cdot 15x^4}{(3x^5 + 3)^2}$$
$$= \frac{-6x^6 + 4x}{3x^{10} + 6x^5 + 3}$$

8) $y = \frac{4x^4 - 2x^2}{5x^4 - 3}$

$$\frac{dy}{dx} = \frac{(5x^4 - 3)(16x^3 - 4x) - (4x^4 - 2x^2) \cdot 20x^3}{(5x^4 - 3)^2}$$
$$= \frac{20x^5 - 48x^3 + 12x}{25x^8 - 30x^4 + 9}$$

9) $y = \dfrac{5x^4 + 3x^3}{5 + 3x^{-5}}$

$$\frac{dy}{dx} = \frac{(5 + 3x^{-5})(20x^3 + 9x^2) - (5x^4 + 3x^3) \cdot -15x^{-6}}{(5 + 3x^{-5})^2}$$
$$= \frac{100x^{13} + 45x^{12} + 135x^8 + 72x^7}{25x^{10} + 30x^5 + 9}$$

For each problem, find the indicated derivative with respect to x.

10) $y = x^5 - 5x^4$ Find $\dfrac{d^3y}{dx^3}$

$$\frac{d^3y}{dx^3} = 60x^2 - 120x$$

11) $y = -4x^5 + 2x^2 + x$ Find $\dfrac{d^4y}{dx^4}$

$$\frac{d^4y}{dx^4} = -480x$$

Differentiate each function with respect to x.

12) $y = ((x^5 - 4)^2 - 4)^4$

$$\frac{dy}{dx} = 4((x^5 - 4)^2 - 4)^3 \cdot 2(x^5 - 4) \cdot 5x^4$$
$$= 40x^4((x^5 - 4)^2 - 4)^3(x^5 - 4)$$

13) $y = (2x - 5)^4 \cdot (5x^4 + 4)^2$

$$\frac{dy}{dx} = (2x - 5)^4 \cdot 2(5x^4 + 4) \cdot 20x^3 + (5x^4 + 4)^2 \cdot 4(2x - 5)^3 \cdot 2$$
$$= 8(2x - 5)^3(5x^4 + 4)(15x^4 - 25x^3 + 4)$$

14) $y = \dfrac{\sqrt[4]{x^5 + 3}}{(-x + 1)^2}$

$$\frac{dy}{dx} = \frac{(-x + 1)^2 \cdot \frac{1}{4}(x^5 + 3)^{-\frac{3}{4}} \cdot 5x^4 - (x^5 + 3)^{\frac{1}{4}} \cdot 2(-x + 1) \cdot -1}{((-x + 1)^2)^2}$$
$$= \frac{3x^5 + 24 + 5x^4}{4(x^5 + 3)^{\frac{3}{4}} \cdot (-x + 1)^3}$$