

## Review Extrema, Rolles and Mean Value Theorem, and Increasing and Decreasing

For each problem, find all points of absolute minima and maxima on the given interval.

1)  $y = -x^3 - 9x^2 - 24x - 15; [-4, -1]$

Absolute minima:  $(-4, 1), (-1, 1)$   
 Absolute maximum:  $(-2, 5)$

2)  $y = -x^2 - 8x - 16; [-6, -3]$

Absolute minimum:  $(-6, -4)$   
 Absolute maximum:  $(-4, 0)$

3)  $y = -x^3 + 2x^2 - 4; (-1, 2)$

Absolute minimum:  $(0, -4)$   
 No absolute maxima.

4)  $y = -x^3 + x^2; (-1, 1)$

Absolute minimum:  $(0, 0)$   
 No absolute maxima.

5)  $y = x^3 - x^2 + 1; [-1, 2]$

Absolute minimum:  $(-1, -1)$   
 No absolute maxima.

6)  $f(x) = -x^3 + 2x^2 + 2; (0, 3]$

Absolute minimum:  $(3, -7)$   
 Absolute maximum:  $\left(\frac{4}{3}, \frac{86}{27}\right)$

7)  $f(x) = -x^4 + 2x^2 - 2; [0, \infty)$

No absolute minima.  
 Absolute maximum:  $(1, -1)$

8)  $y = -x^3 + 9x^2 - 24x + 15; [4, \infty)$

No absolute minima.  
 Absolute maximum:  $(4, -1)$

For each problem, find all points of relative minima and maxima.

9)  $y = -x^4 + x^2 + 3$

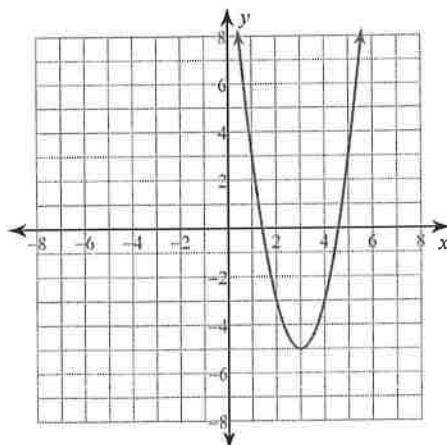
Relative minimum:  $(0, 3)$   
 Relative maxima:  $\left(-\frac{\sqrt{2}}{2}, \frac{13}{4}\right), \left(\frac{\sqrt{2}}{2}, \frac{13}{4}\right)$

10)  $y = -\frac{4x}{x^2 + 4}$

Relative minimum:  $(2, -1)$   
 Relative maximum:  $(-2, 1)$

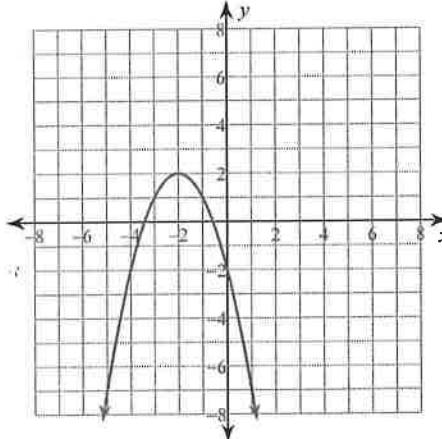
For each problem, find the values of  $c$  that satisfy Rolle's Theorem. Use the provided graph to sketch the function.

11)  $y = 2x^2 - 12x + 13; [2, 4]$



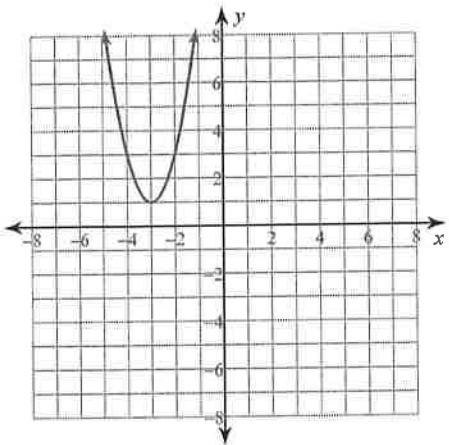
{3}

12)  $y = -x^2 - 4x - 2; [-3, -1]$



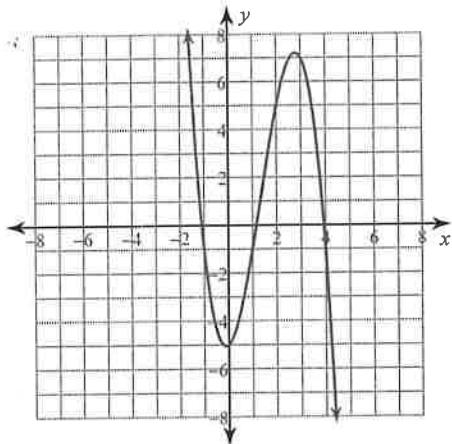
{-2}

13)  $y = 2x^2 + 12x + 19$ ;  $[-4, -2]$



$$\{-3\}$$

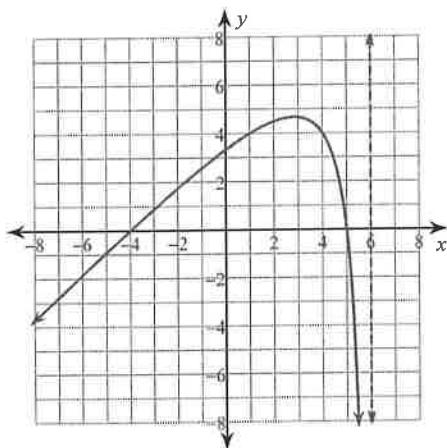
14)  $y = -x^3 + 4x^2 + x - 5$ ;  $[-1, 4]$



$$\left\{ \frac{4 - \sqrt{19}}{3}, \frac{4 + \sqrt{19}}{3} \right\}$$

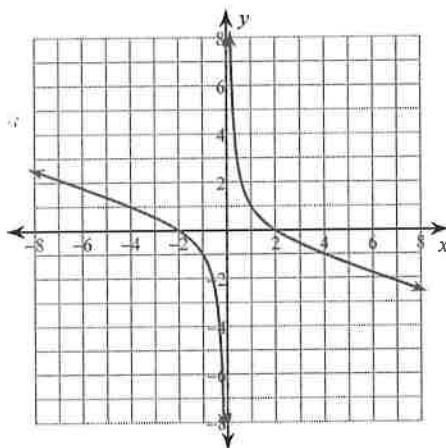
For each problem, determine if Rolle's Theorem can be applied. If it can, find all values of  $c$  that satisfy the theorem. If it cannot, explain why not. Use the provided graph to sketch the function.

15)  $y = \frac{-x^2 + x + 20}{-x + 6}$ ;  $[-4, 5]$



$$\{6 - \sqrt{10}\}$$

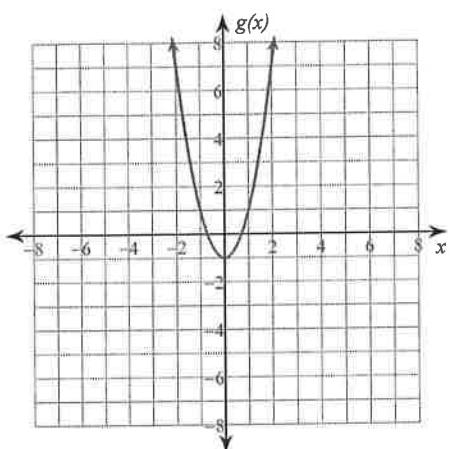
16)  $y = \frac{-x^2 + 4}{3x}$ ;  $[-2, 2]$



The function is not continuous on  $[-2, 2]$

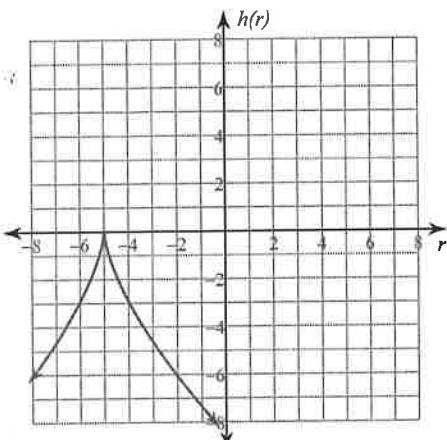
For each problem, find the values of  $c$  that satisfy the Mean Value Theorem. Use the provided graph to sketch the function.

17)  $g(x) = 2x^2 - 1; [-2, 0]$



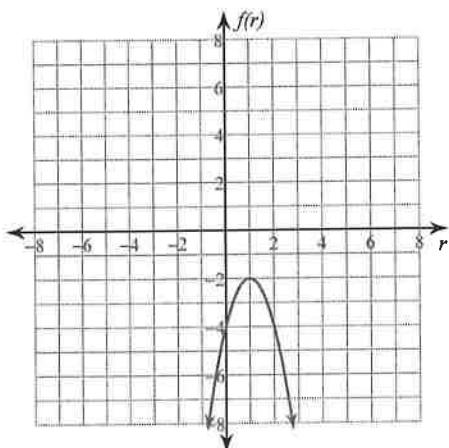
$$\{-1\}$$

18)  $h(r) = -(5r + 25)^{\frac{2}{3}}; [-5, -3]$



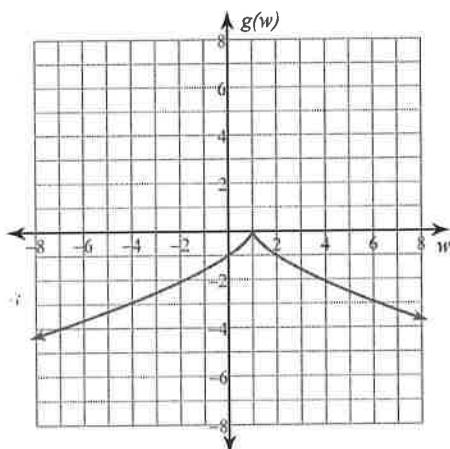
$$\left\{-\frac{119}{27}\right\}$$

19)  $f(r) = -2r^2 + 4r - 4; [0, 2]$



$$\{1\}$$

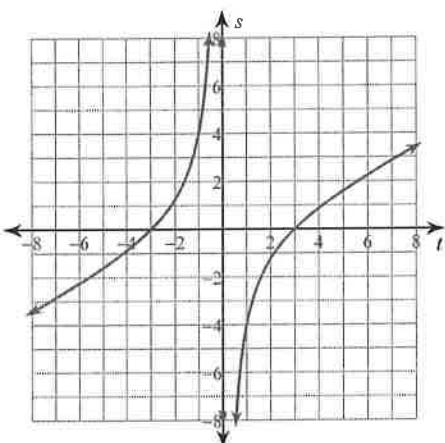
20)  $g(w) = -(w - 1)^{\frac{2}{3}}; [1, 5]$



$$\left\{\frac{59}{27}\right\}$$

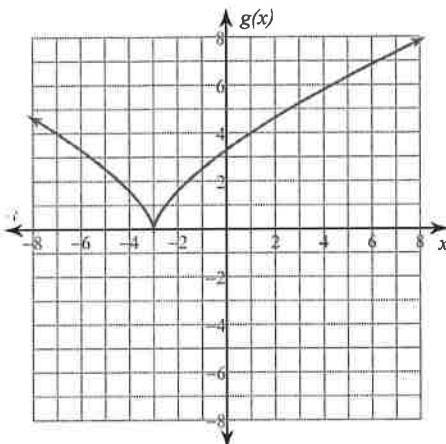
For each problem, determine if the Mean Value Theorem can be applied. If it can, find all values of  $c$  that satisfy the theorem. If it cannot, explain why not. Use the provided graph to sketch the function.

21)  $s = \frac{t^2 - 9}{2t}$ ;  $[1, 6]$



$$\{\sqrt{6}\}$$

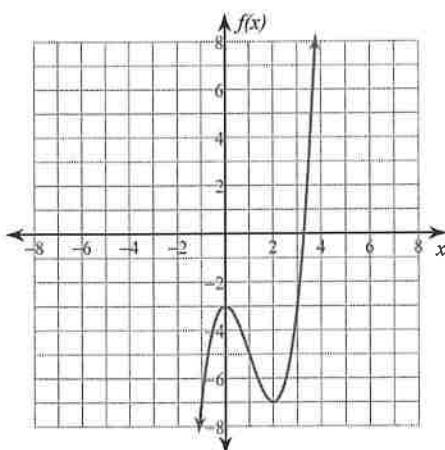
22)  $g(x) = (2x + 6)^{\frac{2}{3}}$ ;  $[-4, -1]$



The function is not differentiable on  $(-4, -1)$

For each problem, find the x-coordinates of all critical points, find all discontinuities, and find the open intervals where the function is increasing and decreasing. Use the provided graph to sketch the function.

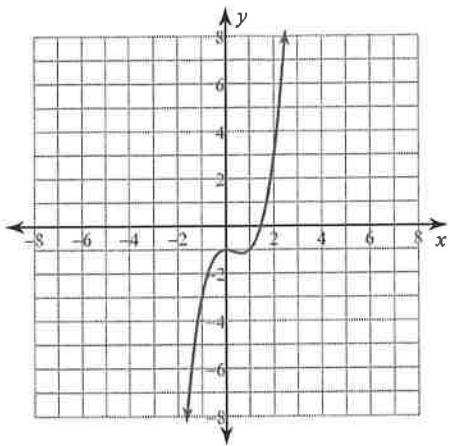
23)  $f(x) = x^3 - 3x^2 - 3$



Critical points at:  $x = 0, 2$  No discontinuities exist.

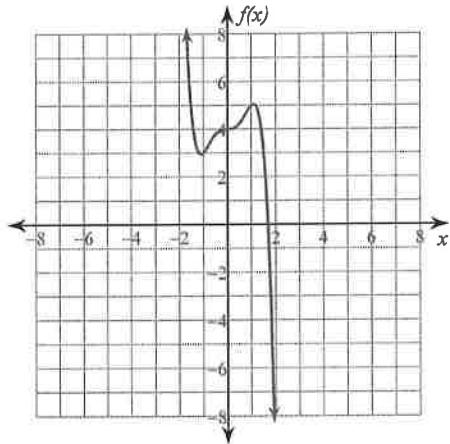
Increasing:  $(-\infty, 0), (2, \infty)$  Decreasing:  $(0, 2)$

24)  $y = x^3 - x^2 - 1$



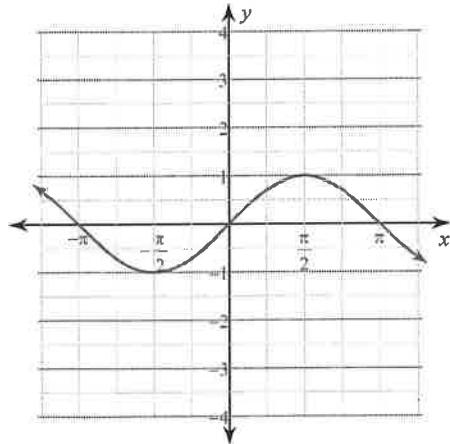
Critical points at:  $x = 0, \frac{2}{3}$  No discontinuities exist  
 Increasing:  $(-\infty, 0), \left(\frac{2}{3}, \infty\right)$  Decreasing:  $\left(0, \frac{2}{3}\right)$

25)  $f(x) = -x^5 + 2x^3 + 4$



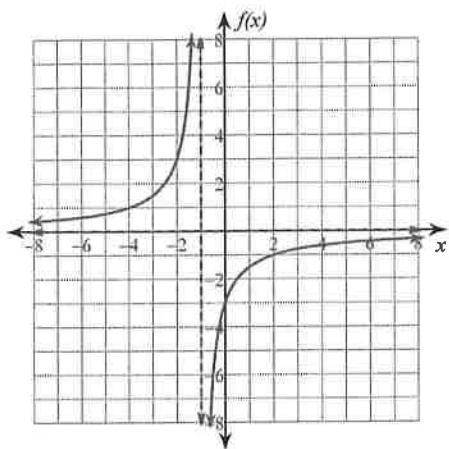
Critical points at:  $x = -\frac{\sqrt{30}}{5}, 0, \frac{\sqrt{30}}{5}$  No di  
 Increasing:  $\left(-\frac{\sqrt{30}}{5}, \frac{\sqrt{30}}{5}\right)$  Decreasing:  $\left(-\infty, -\frac{\sqrt{30}}{5}\right) \cup \left(\frac{\sqrt{30}}{5}, \infty\right)$

26)  $y = \sin(x); [-\pi, \pi]$



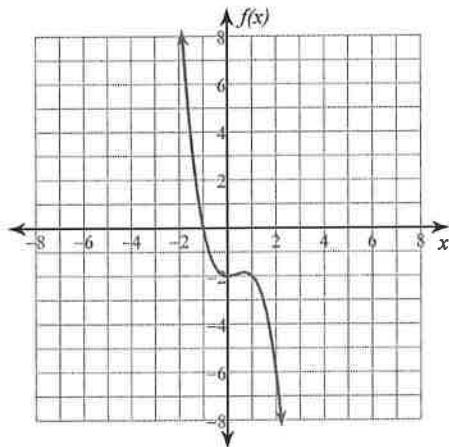
Critical points at:  $x = -\frac{\pi}{2}, \frac{\pi}{2}$  No discontinuities  
 Increasing:  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$  Decreasing:  $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

27)  $f(x) = -\frac{3}{x+1}$



No critical points exist. Discontinuity at:  $x = -1$   
 Increasing:  $(-\infty, -1), (-1, \infty)$  Decreasing: No intervals exist.

28)  $f(x) = -x^3 + x^2 - 2$



Critical points at:  $x = 0, \frac{2}{3}$  No discontinuities exist.

Increasing:  $\left(0, \frac{2}{3}\right)$  Decreasing:  $(-\infty, 0), \left(\frac{2}{3}, \infty\right)$