MATH 2413 - Calculus I
Sigma Notation/Summation Exercises

Some Relevant Formulas and Properties

$$\sum_{j=m}^{n} kf(j) = k\sum_{j=m}^{n} f(j) \qquad \sum_{j=m}^{n}[f(j) \pm g(j)] = \sum_{j=m}^{n} f(j) \pm \sum_{j=m}^{n} g(j) \qquad \sum_{j=1}^{n} c = nc$$

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2} \qquad \sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{j=1}^{n} j^3 = \left[\frac{n(n+1)}{2}\right]^2$$

In Exercises 1-16, evaluate the given summation.

1. $\sum_{j=0}^{3} 2^j$
2. $\sum_{j=0}^{3} j^2$
3. $\sum_{j=1}^{5} (2j-3)$
4. $\sum_{j=3}^{6} (-1)^j$
5. $\sum_{j=0}^{4} \frac{24}{j!}$
6. $\sum_{j=1}^{4} \frac{(-1)^j}{j}$
7. $\sum_{j=1}^{5} 3$
8. $\sum_{j=1}^{1000} 7$
9. $\sum_{j=1}^{100} j$
10. $\sum_{j=1}^{50} j^2$
11. $\sum_{j=1}^{1000} 5j$
12. $\sum_{j=1}^{100} -7j^2$
13. $\sum_{j=1}^{500} \left(3j^2 - 5j + 2\right)$
14. $\sum_{j=1}^{90} \left(-2j^2 + 3j - 5\right)$
15. $\sum_{j=1}^{100} (2j-3)^2$
16. $\sum_{j=1}^{100} (-j+2)^2$

In Exercises 17-20, express each summation in a simplified algebraic expression that does not use sigma notation.

17. $\sum_{j=1}^{n} (3j-2)$
18. $\sum_{j=1}^{n} (3j-4)^2$
19. $\sum_{j=1}^{n} \left[(3j-5)\cdot\frac{1}{n^2}\right]$
20. $\sum_{j=1}^{n} \left[(j-2)^2\cdot\frac{1}{n^3}\right]$

In Exercises 21-22, evaluate the given expression.

21. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[(3j-5)\cdot\frac{1}{n^2}\right]$
22. $\lim_{n\to\infty} \sum_{j=1}^{n} \left[(j-2)^2\cdot\frac{1}{n^3}\right]$

23. Find the exact area under $f(x) = 3x + 5$ from a=0 to b=3 by using an infinite summation.

24. Find the exact area under $f(x) = 2x^2 + 8$ from a=1 to b=3 by using an infinite summation.

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5) $\sum_{j=0}^{4} \frac{24}{j!} = \frac{24}{1} + \frac{24}{1} + \frac{24}{2} + \frac{24}{6} + \frac{24}{24} = 65$

13) $\sum_{j=1}^{500} (3j^2 - 5j + 2) = 3\sum_{j=1}^{500} j^2 - 5\sum_{j=1}^{500} j + \sum_{j=1}^{500} 2$

$= 3\left(\frac{500(501)(1001)}{6}\right) - 5\left(\frac{500(501)}{2}\right) + 500(2)$

$= 124{,}750{,}000$

15) $\sum_{j=1}^{100} (2j-3)^2 = \sum_{j=1}^{100} 4j^2 - 12j + 9$

$= 4\left(\frac{100(101)(201)}{6}\right) - 12\left(\frac{100(101)}{2}\right) + 9(100)$

$= 1{,}293{,}700$

20) $\sum_{j=1}^{n} \left[(j-2)^2 \cdot \frac{1}{n^3}\right] = \frac{1}{n^3} \sum_{j=1}^{n} j^2 - 4j + 4$

$= \frac{1}{n^{\cancel{3}2}}\left[\left(\frac{\cancel{n}(n+1)(2n+1)}{6}\right) - \cancel{4}^{2}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right) + 4\cancel{n}\right]$