Calculus

Name_____

Review of Trig, Trig Inverse, Ln, and E

Date_____ Period____

Differentiate each function with respect to x.

1) $y = \csc^{-1}(-x^6 + 10)^{10}$

$$\frac{dy}{dx} = -\frac{1}{\left|(-x^6+10)^{10}\right|\sqrt{\left((-x^6+10)^{10}\right)^2 - 1}} \cdot 10(-x^6+10)^9 \cdot -6x^5$$

$$= \frac{60x^5}{\sqrt{(-x^6+10)^{20} - 1}(-x^6+10)}$$

2) $y = \cos^{-1}(3x^7+1)^3$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1 - \left((3x^7+1)^3\right)^2}} \cdot 3(3x^7+1)^2 \cdot 21x^6$$

$$= -\frac{63x^6(3x^7+1)^2}{\sqrt{1-(3x^7+1)^6}}$$

3) $y = \left(\cot^{-1} 3x^5\right)^6$

$$\frac{dy}{dx} = 6\left(\cot^{-1} 3x^5\right)^5 \cdot -\frac{1}{(3x^5)^2+1} \cdot 15x^4$$

$$= -\frac{90x^4\left(\cot^{-1} 3x^5\right)^5}{9x^{10}+1}$$

4) $y = \sin^{-1}(7x^2-9)^7$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - \left((7x^2-9)^7\right)^2}} \cdot 7(7x^2-9)^6 \cdot 14x$$

$$= \frac{98x(7x^2-9)^6}{\sqrt{1-(7x^2-9)^{14}}}$$

5) $y = \sec^{-1} -8x^9$

$$\frac{dy}{dx} = \frac{1}{\left|-8x^9\right|\sqrt{(-8x^9)^2 - 1}} \cdot -72x^8$$

$$= -\frac{72x^8}{\left|-8x^9\right|\sqrt{64x^{18} - 1}}$$

6) $y = \tan^{-1}(x^5-9)^3$

$$\frac{dy}{dx} = \frac{1}{\left((x^5-9)^3\right)^2 + 1} \cdot 3(x^5-9)^2 \cdot 5x^4$$

$$= \frac{15x^4(x^5-9)^2}{(x^5-9)^6+1}$$

7) $y = \ln 4x^3$

$$\frac{dy}{dx} = \frac{1}{4x^3} \cdot 12x^2$$

$$= \frac{3}{x}$$

8) $y = e^{x^3}$

$$\frac{dy}{dx} = e^{x^3} \cdot 3x^2$$

9) $y = \ln\left(\frac{2x^5}{x^3+5}\right)^3$

$$\frac{dy}{dx} = 3\left(\frac{1}{2x^5}\cdot 10x^4 - \frac{1}{x^3+5}\cdot 3x^2\right)$$
$$= \frac{3(2x^3+25)}{x(x^3+5)}$$

10) $y = e^{\sqrt[4]{3x^5-1}}$

$$\frac{dy}{dx} = e^{(3x^5-1)^{\frac{1}{4}}}\cdot\frac{1}{4}(3x^5-1)^{-\frac{3}{4}}\cdot 15x^4$$
$$= \frac{15x^4 e^{\sqrt[4]{3x^5-1}}}{4\sqrt[4]{(3x^5-1)^3}}$$

11) $y = \ln\sqrt[4]{\frac{5x^3}{2x^2+1}}$

$$\frac{dy}{dx} = \frac{1}{4}\left(\frac{1}{5x^3}\cdot 15x^2 - \frac{1}{2x^2+1}\cdot 4x\right)$$
$$= \frac{2x^2+3}{4x(2x^2+1)}$$

12) $y = \frac{e^{3x^4}}{e^{4x^3-5}}$

$$\frac{dy}{dx} = e^{3x^4-(4x^3-5)}(12x^3-12x^2)$$
$$= 12x^2 e^{3x^4-4x^3+5}(x-1)$$

13) $y = \sec(\csc 4x^2)$

$$\frac{dy}{dx} = \sec(\csc 4x^2)\tan(\csc 4x^2)\cdot -\csc 4x^2\cot 4x^2\cdot 8x$$
$$= -8x\sec(\csc 4x^2)\tan(\csc 4x^2)\csc 4x^2\cot 4x^2$$

14) $y = \frac{\tan 3x^4}{\sec 3x^3}$

$$\frac{dy}{dx} = \frac{\sec 3x^3\cdot\sec^2 3x^4\cdot 12x^3 - \tan 3x^4\cdot\sec 3x^3\tan 3x^3\cdot 9x^2}{\sec^2 3x^3}$$
$$= \frac{3x^2(4x\sec^2 3x^4 - 3\tan 3x^4\tan 3x^3)}{\sec 3x^3}$$

Evaluate each indefinite integral.

15) $\int \frac{1}{\sqrt{9-x^2}}\,dx$

$\sin^{-1}\frac{x}{3} + C$

16) $\int \frac{1}{4+x^2}\,dx$

$\frac{1}{2}\cdot\tan^{-1}\frac{x}{2} + C$

17) $\int \frac{e^x}{e^x\sqrt{e^{2x}-4}}\,dx$

$\frac{1}{2}\cdot\sec^{-1}\frac{|e^x|}{2} + C$

18) $\int \frac{e^x}{4+e^{2x}}\,dx$

$\frac{1}{2}\cdot\tan^{-1}\frac{e^x}{2} + C$

19) $\int \frac{1}{x\left(4+(\ln 5x)^2\right)}\,dx$

$\frac{1}{2}\cdot\tan^{-1}\frac{\ln 5x}{2}+C$

20) $\int \frac{6x^2}{\sqrt{16-4x^6}}\,dx$

$\sin^{-1}\frac{2x^3}{4}+C$

21) $\int \frac{2x}{1+x^4}\,dx$

$\tan^{-1}x^2+C$

22) $\int \frac{1}{x\left(9+(\ln 2x)^2\right)}\,dx$

$\frac{1}{3}\cdot\tan^{-1}\frac{\ln 2x}{3}+C$

23) $\int 5x^{-1}\,dx$

$5\ln|x|+C$

24) $\int -\frac{5}{x}\,dx$

$-5\ln|x|+C$

25) $\int -10e^x\,dx$

$-10e^x+C$

26) $\int \frac{75x^2}{5x^3+1}\,dx$

$5\ln|5x^3+1|+C$

27) $\int \frac{10\cos 5x}{\sin 5x}\,dx$

$2\ln|\sin 5x|+C$

28) $\int \frac{2}{x(-4+\ln -5x)}\,dx$

$2\ln|-4+\ln -5x|+C$

29) $\int -\frac{8e^{2x}}{e^{2x}-4}\,dx$

$-4\ln|e^{2x}-4|+C$

30) $\int -\frac{4\sec^2 x}{\tan x}\,dx$

$-4\ln|\tan x|+C$