

Integration of Bases other than e

I. Integration for Bases other than e .

$$A. \int a^x dx = \frac{1}{\ln a} a^x + C$$

$$B. \int a^u du = \frac{1}{\ln a} a^u \cdot u' + C$$

Evaluate each indefinite integral.

$$1. \int 5^x dx$$

$$= \frac{1}{\ln(5)} 5^x + C$$

$$3. \int 4x^2(5^{-x^3})dx$$

$$\begin{aligned} u &= -x^3 \\ du &= -3x^2 dx \end{aligned}$$

$$\begin{aligned} &-\frac{1}{3} \int -3x^2(5^{-x^3})dx \\ &= -\frac{4}{3} \int 5^u du = \frac{1}{\ln 5} 5^u + C \\ &= \frac{1}{\ln 5} 5^{-x^3} + C = \boxed{\frac{1}{\ln(5)} 5^{-x^3} + C} \end{aligned}$$

$$2. \int 2x(3^{x^2})dx$$

$$\begin{aligned} u &= x^2 \\ du &= 2x dx \end{aligned}$$

$$\int 3^u du = \frac{1}{\ln 3} 3^x + C = \boxed{\frac{3^x}{\ln 3} + C}$$

$$4. \int \frac{1}{7^x} \int \frac{\ln 7^x}{7^x+4} dx$$

$$\begin{aligned} u &= 7^x + 4 \\ du &= \ln 7 \cdot 7^x \end{aligned}$$

$$\begin{aligned} &= \frac{1}{\ln 7} \int \frac{1}{u} du \\ &= \frac{1}{\ln 7} (\ln|u|) + C \\ &= \boxed{\frac{\ln|7^x+4|}{\ln 7} + C} \end{aligned}$$

$$5. \int \frac{8^{7x}}{1+8^{7x}} dx$$

$$\begin{aligned} u &= 1+8^{7x} \\ du &= \ln 8 \cdot 8^{7x} \cdot 7 \\ &= 7 \ln 8 \cdot 8^{7x} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{7 \ln 8} \int \frac{7 \ln(8) \cdot 8^{7x}}{1+8^{7x}} dx \\ &= \frac{1}{7 \ln 8} \int \frac{1}{u} du \end{aligned}$$

$$\begin{aligned} &= \frac{1}{7 \ln 8} \ln|1+8^{7x}| + C \\ &= \boxed{\frac{\ln|1+8^{7x}|}{7 \ln 8} + C} \end{aligned}$$

6. Solve the differential equation with the given initial condition.

$$\frac{dy}{dx} = 3e^x + 5 \sin x ; \quad f(0) = 2$$

$$dy = (3e^x + 5 \sin x) dx$$

$$\int dy = \int (3e^x + 5 \sin x) dx$$

$$y + C_1 = 3e^x - 5 \cos x + C_2$$

solve for C
when $x=0$
 $y=0$

$$y = 3e^x - 5 \cos x + C$$

$$2 = 3(1) - 5(1) + C$$

$$2 = 3(1) - 5 + C$$

$$2 = -2 + C$$

$$y = 3e^x - 5 \cos(x) + 4$$

Derivatives of Bases other than e

I. Derivatives for Bases Other than e

Let a be a positive number ($a \neq 1$) and let u be a differential function of x .

A. $\frac{d}{dx}(a^x) = (\ln a) a^x$

B. $\frac{d}{dx}(a^u) = (\ln a) a^u \cdot u'$

C. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

D. $\frac{d}{dx}(\log_a u) = \frac{1}{u \ln a} \cdot u'$

Find the derivative of each function.

1. $f(x) = 5^x$

2. $g(x) = 12^{2-3x}$

3. $y = x^3 3^x$ product rule

$$y' = \ln(5) \cdot 5^x$$

$$y' = (\ln 12) 12^{2-3x} \cdot (-3)$$

$$y' = 3x^2 \cdot 3^x + x^3 \ln(3) 3^x$$

4. $f(x) = 7^\theta \sin 5\theta$

Product rule

5. $y = \log_8 x$

6. $y = \log_3 \frac{x^5}{x+4}$ chain rule w/ quotient rule

$$y' = \frac{1}{\frac{x^5}{x+4} \ln 3} \cdot \left(\frac{5x^4(x+4) - 1(x^5)}{(x+4)^2} \right)$$

$$= \frac{x+4}{x^5 \ln 3} \left(\frac{5x^5 + 20x^4 - x^5}{(x+4)^2} \right)$$

$$= \frac{x+4}{x^5 \ln 3} \left(\frac{4x^5 + 20x^4}{(x+4)^2} \right)$$

Simplified
for enough

7. Use logarithmic differentiation to find $\frac{dy}{dx}$.

$$y = (2x+1)^x$$

$$\ln y = \ln(2x+1)^x$$

$$\ln y = x \cdot \ln(2x+1)$$

$$y' = 1 \cdot \ln(2x+1) + x \cdot \frac{1}{2x+1} \cdot 2$$

$$y' = \left(\ln(2x+1) + \frac{2x}{2x+1} \right) y$$

$$y' = \left(\ln(2x+1) + \frac{2x}{2x+1} \right) (2x+1)^x$$