Exercises for Section 5.9

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–6, evaluate the function. If the value is not a rational number, round your answer to three decimal places.

1. (a) $\sinh 3$
   (b) $\tanh(-2)$
2. (a) $\cosh 0$
   (b) $\operatorname{sech} 1$
3. (a) $\operatorname{csch}(\ln 2)$
   (b) $\coth(\ln 5)$
4. (a) $\sinh^{-1} 0$
   (b) $\tanh^{-1} 0$
5. (a) $\cosh^{-1} 2$
   (b) $\operatorname{sech}^{-1} \frac{2}{3}$
6. (a) $\operatorname{csch}^{-1} 2$
   (b) $\coth^{-1} 3$

In Exercises 7–12, verify the identity.

7. $\tanh^2 x + \operatorname{sech}^2 x = 1$
8. $\cosh^2 x = \dfrac{1 + \cosh 2x}{2}$
9. $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$
10. $\sinh 2x = 2 \sinh x \cosh x$
11. $\sinh 3x = 3 \sinh x + 4 \sinh^3 x$
12. $\cosh x + \cosh y = 2 \cosh \dfrac{x + y}{2} \cosh \dfrac{x - y}{2}$

In Exercises 13 and 14, use the value of the given hyperbolic function to find the values of the other hyperbolic functions at x.

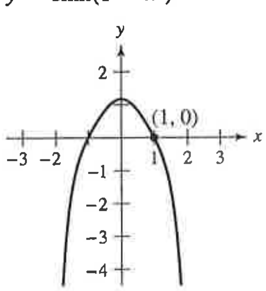
13. $\sinh x = \dfrac{3}{2}$
14. $\tanh x = \dfrac{1}{2}$

In Exercises 15–24, find the derivative of the function.

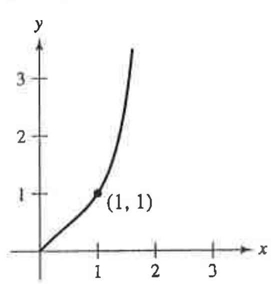
15. $y = \operatorname{sech}(x + 1)$
16. $y = \coth 3x$
17. $f(x) = \ln(\sinh x)$
18. $g(x) = \ln(\cosh x)$
19. $y = \ln\left(\tanh \dfrac{x}{2}\right)$
20. $y = x \cosh x - \sinh x$
21. $h(x) = \dfrac{1}{4} \sinh 2x - \dfrac{x}{2}$
22. $h(t) = t - \coth t$
23. $f(t) = \arctan(\sinh t)$
24. $g(x) = \operatorname{sech}^2 3x$

In Exercises 25–28, find an equation of the tangent line to the graph of the function at the given point.

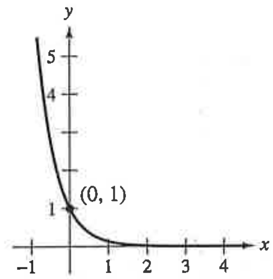
25. $y = \sinh(1 - x^2)$



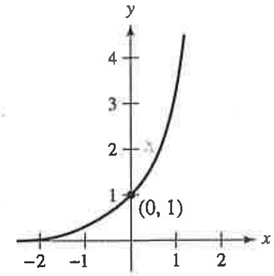
26. $y = x^{\cosh x}$



27. $y = (\cosh x - \sinh x)^2$



28. $y = e^{\sinh x}$



In Exercises 29–32, find any relative extrema of the function. Use a graphing utility to confirm your result.

29. $f(x) = \sin x \sinh x - \cos x \cosh x, \quad -4 \le x \le 4$
30. $f(x) = x \sinh(x - 1) - \cosh(x - 1)$
31. $g(x) = x \operatorname{sech} x$
32. $h(x) = 2 \tanh x - x$

In Exercises 33 and 34, show that the function satisfies the differential equation.

	Function	*Differential Equation*
33.	$y = a \sinh x$	$y''' - y' = 0$
34.	$y = a \cosh x$	$y'' - y = 0$

Linear and Quadratic Approximations **In Exercises 35 and 36, use a computer algebra system to find the linear approximation**

$$P_1(x) = f(a) + f'(a)(x - a)$$

and the quadratic approximation

$$P_2(x) = f(a) + f'(a)(x - a) + \tfrac{1}{2}f''(a)(x - a)^2$$

of the function f at $x = a$. Use a graphing utility to graph the function and its linear and quadratic approximations.

35. $f(x) = \tanh x, \quad a = 0$
36. $f(x) = \cosh x, \quad a = 0$

Catenary **In Exercises 37 and 38, a model for a power cable suspended between two towers is given. (a) Graph the model, (b) find the heights of the cable at the towers and at the midpoint between the towers, and (c) find the slope of the model at the point where the cable meets the right-hand tower.**

37. $y = 10 + 15 \cosh \dfrac{x}{15}, \quad -15 \le x \le 15$
38. $y = 18 + 25 \cosh \dfrac{x}{25}, \quad -25 \le x \le 25$

In Exercises 39–50, find the integral.

39. $\displaystyle\int \sinh(1 - 2x)\, dx$
40. $\displaystyle\int \frac{\cosh \sqrt{x}}{\sqrt{x}}\, dx$
41. $\displaystyle\int \cosh^2(x - 1) \sinh(x - 1)\, dx$
42. $\displaystyle\int \frac{\sinh x}{1 + \sinh^2 x}\, dx$

43. $\displaystyle\int \frac{\cosh x}{\sinh x}\,dx$

44. $\displaystyle\int \operatorname{sech}^2(2x - 1)\,dx$

45. $\displaystyle\int x \operatorname{csch}^2 \frac{x^2}{2}\,dx$

46. $\displaystyle\int \operatorname{sech}^3 x \tanh x\,dx$

47. $\displaystyle\int \frac{\operatorname{csch}(1/x)\coth(1/x)}{x^2}\,dx$

48. $\displaystyle\int \frac{\cosh x}{\sqrt{9 - \sinh^2 x}}\,dx$

49. $\displaystyle\int \frac{x}{x^4 + 1}\,dx$

50. $\displaystyle\int \frac{2}{x\sqrt{1 + 4x^2}}\,dx$

In Exercises 51–56, evaluate the integral.

51. $\displaystyle\int_0^{\ln 2} \tanh x\,dx$

52. $\displaystyle\int_0^1 \cosh^2 x\,dx$

53. $\displaystyle\int_0^4 \frac{1}{25 - x^2}\,dx$

54. $\displaystyle\int_0^4 \frac{1}{\sqrt{25 - x^2}}\,dx$

55. $\displaystyle\int_0^{\sqrt{2}/4} \frac{2}{\sqrt{1 - 4x^2}}\,dx$

56. $\displaystyle\int_0^{\ln 2} 2e^{-x}\cosh x\,dx$

In Exercises 57–64, find the derivative of the function.

57. $y = \cosh^{-1}(3x)$

58. $y = \tanh^{-1}\dfrac{x}{2}$

59. $y = \sinh^{-1}(\tan x)$

60. $y = \operatorname{sech}^{-1}(\cos 2x),\quad 0 < x < \pi/4$

61. $y = \tanh^{-1}(\sin 2x)$

62. $y = (\operatorname{csch}^{-1} x)^2$

63. $y = 2x\sinh^{-1}(2x) - \sqrt{1 + 4x^2}$

64. $y = x\tanh^{-1} x + \ln\sqrt{1 - x^2}$

Writing About Concepts

65. Discuss several ways in which the hyperbolic functions are similar to the trigonometric functions.

66. Sketch the graph of each hyperbolic function. Then identify the domain and range of each function.

Limits **In Exercises 67–72, find the limit.**

67. $\displaystyle\lim_{x\to\infty} \sinh x$

68. $\displaystyle\lim_{x\to\infty} \tanh x$

69. $\displaystyle\lim_{x\to\infty} \operatorname{sech} x$

70. $\displaystyle\lim_{x\to-\infty} \operatorname{csch} x$

71. $\displaystyle\lim_{x\to 0} \frac{\sinh x}{x}$

72. $\displaystyle\lim_{x\to 0^-} \coth x$

In Exercises 73–80, find the indefinite integral using the formulas of Theorem 5.24.

73. $\displaystyle\int \frac{1}{\sqrt{1 + e^{2x}}}\,dx$

74. $\displaystyle\int \frac{x}{9 - x^4}\,dx$

75. $\displaystyle\int \frac{1}{\sqrt{x}\sqrt{1 + x}}\,dx$

76. $\displaystyle\int \frac{\sqrt{x}}{\sqrt{1 + x^3}}\,dx$

77. $\displaystyle\int \frac{-1}{4x - x^2}\,dx$

78. $\displaystyle\int \frac{dx}{(x + 2)\sqrt{x^2 + 4x + 8}}$

79. $\displaystyle\int \frac{1}{1 - 4x - 2x^2}\,dx$

80. $\displaystyle\int \frac{dx}{(x + 1)\sqrt{2x^2 + 4x + 8}}$

In Exercises 81–84, solve the differential equation.

81. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{80 + 8x - 16x^2}}$

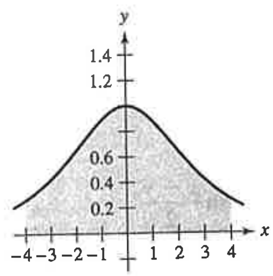
82. $\dfrac{dy}{dx} = \dfrac{1}{(x - 1)\sqrt{-4x^2 + 8x - 1}}$

83. $\dfrac{dy}{dx} = \dfrac{x^3 - 21x}{5 + 4x - x^2}$

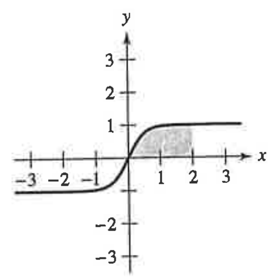
84. $\dfrac{dy}{dx} = \dfrac{1 - 2x}{4x - x^2}$

Area **In Exercises 85–88, find the area of the region.**

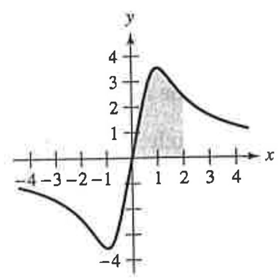
85. $y = \operatorname{sech}\dfrac{x}{2}$



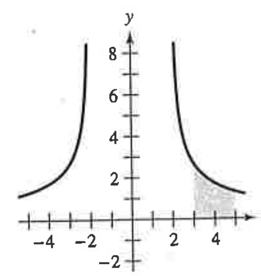
86. $y = \tanh 2x$



87. $y = \dfrac{5x}{\sqrt{x^4 + 1}}$



88. $y = \dfrac{6}{\sqrt{x^2 - 4}}$



In Exercises 89 and 90, evaluate the integral in terms of (a) natural logarithms and (b) inverse hyperbolic functions.

89. $\displaystyle\int_0^{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 1}}$

90. $\displaystyle\int_{-1/2}^{1/2} \frac{dx}{1 - x^2}$

91. ***Chemical Reactions*** Chemicals A and B combine in a 3-to-1 ratio to form a compound. The amount of compound x being produced at any time t is proportional to the unchanged amounts of A and B remaining in the solution. So, if 3 kilograms of A is mixed with 2 kilograms of B, you have

$$\frac{dx}{dt} = k\left(3 - \frac{3x}{4}\right)\left(2 - \frac{x}{4}\right) = \frac{3k}{16}(x^2 - 12x + 32).$$

One kilogram of the compound is formed after 10 minutes. Find the amount formed after 20 minutes by solving the equation

$$\int \frac{3k}{16}\,dt = \int \frac{dx}{x^2 - 12x + 32}.$$