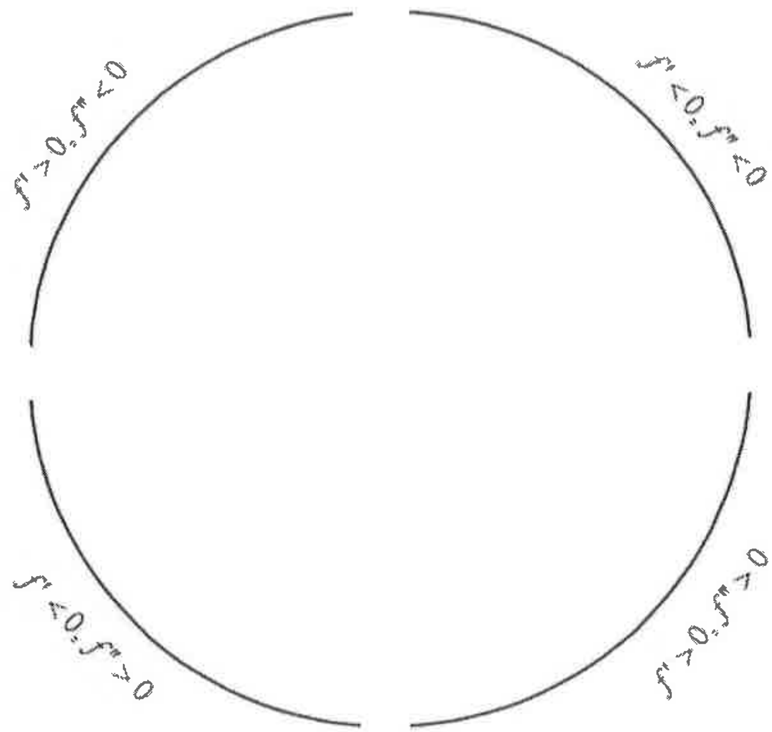
Curve Sketching Summary

For a function f, the combined information of the first derivative f' and the second derivative f'' can tell us the shape of a graph.



CRITICAL VALUES

- If $f(c)$ is defined and either $f'(c) = 0$ or $f'(c) = DNE$, then $x = c$ is a critical value of f.

Note: A critical value can occur at a discontinuity, as long as f is defined at $x = c$.

INCREASING/DECREASING TEST

- If $f'(x) > 0$ on an interval, then f is increasing on that interval.
- If $f'(x) < 0$ on an interval, then f is increasing on that interval.

Note: A function can change its increasing/decreasing behavior at a critical value OR a discontinuity.

POSSIBLE INFLECTION VALUES

- If $f(c)$ is defined and either $f''(c) = 0$ or $f''(c) = DNE$, then $x = c$ is a possible inflection value of f.

Note: A possible inflection value can occur at a discontinuity, as long as f is defined at $x = c$.

CONCAVITY TEST

- If $f''(x) > 0$ on an interval, then f is concave up on that interval.
- If $f''(x) < 0$ on an interval, then f is concave down on that interval.

Note: A function can change its concavity at a possible inflection value OR a discontinuity.

FIRST DERIVATIVE TEST for relative extrema (local argument)

Suppose that $x=c$ is a critical value of f.

- If f' changes from positive to negative at $x=c$, then f has a relative maximum of $f(c)$ at $x=c$.
- If f' changes from negative to positive at $x=c$, then f has a relative minimum of $f(c)$ at $x=c$.

NOTE: f must be continuous at $x=c$ AND not all critical values yield relative extrema.

SECOND DERIVATIVE TEST for relative extrema (local argument)

- If $f'(c)=0$ and $f''(c)<0$, then f has a relative maximum of $f(c)$ at $x=c$.
- If $f'(c)=0$ and $f''(c)>0$, then f has a relative minimum of $f(c)$ at $x=c$.

NOTE: f must be twice-differentiable at $x=c$ AND $f''(c)=0$ is inconclusive.

POINT OF INFLECTION TEST

Suppose that $x=c$ is a possible inflection value of f.

- If f'' changes from either positive to negative OR negative to positive at $x=c$, then f has an inflection point at $x=c$.

FIRST DERIVATIVE TEST for absolute extrema (global argument)

Suppose that $x=c$ is a critical value of f.

- If $f'>0$ **for all** $x<c$ and $f'<0$ **for all** $x>c$, then f has an absolute maximum of $f(c)$ at $x=c$.
- If $f'<0$ **for all** $x<c$ and $f'>0$ **for all** $x>c$, then f has an absolute minimum of $f(c)$ at $x=c$.

SECOND DERIVATIVE TEST for absolute extrema (global argument)

- If $f'(c)=0$ and $f''(c)<0$ **for all** x, then f has an absolute maximum of $f(c)$ at $x=c$.
- If $f'(c)=0$ and $f''(c)>0$ **for all** x, then f has an absolute minimum of $f(c)$ at $x=c$.

CLOSED INTERVAL TEST for absolute extrema

For a continuous function f on a closed interval $[a,b]$:

1) Find the values of f at the endpoint of the interval, that is, find $f(a)$ and $f(b)$.
2) Find all critical values of f **in the open interval** (a,b).
3) Find the values of f at each of the critical values **in the open interval** (a,b).
4) The largest value of f is the absolute maximum, and the smallest value of f is the absolute minimum value.

Note: You may find critical values of f that are not in the open interval (a,b). While these will certainly be critical values of f, they are not included in the test if they are not in the open interval (a,b).

GUIDELINES FOR CURVE SKETCHING

1) Domain 2) Discontinuities 3) Symmetry 4) End Behavior 5) Intercepts 6) Increasing/Decreasing 7) Relative Extrema 8) Concavity 9) Inflection Points 10) Plug in carefully chosen x-values judiciously

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A LAST IMPORTANT REMINDER TO INCULCATE AND REITERATE

An <u>extrema</u>, whether a Relative Max/Relative Min or Absolute Max/Absolute Min is the ***y*-value**. The <u>location</u> of the extrema is the ***x*-value**.