

Review integration

Evaluate each sum.

1) $\sum_{k=1}^n (8k + 4)$

$4n^2 + 8n$

2) $\sum_{k=1}^n (16k + 16)$

$8n^2 + 24n$

Evaluate each limit.

3) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{6}{n} + \frac{27k}{n^2} \right)$

$\frac{39}{2} = 19.5$

4) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{n} + \frac{2k}{n^2} \right)$

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For each problem, find the area under the curve over the given interval. Set up your solution using the limit as n goes to ∞ of the upper sum. and by 4 rectangles

5) $f(x) = 2x^2 + 4; [1, 2]$

$\frac{26}{3} \approx 8.667$

6) $f(x) = x^2 + 6; [1, 2]$

$\frac{25}{3} \approx 8.333$

For each problem, find the area under the curve over the given interval. Set up your solution using the limit as n goes to ∞ of the lower sum. and by 4 rectangles

7) $f(x) = 2x^2 + 5; [0, 2]$

$\frac{46}{3} \approx 15.333$

8) $f(x) = 2x^2 + 2; [0, 1]$

$\frac{8}{3} \approx 2.667$

Express each definite integral in terms of u , but do not evaluate.

9) $\int_{-2}^1 \frac{12x}{(2x^2 + 1)^2} dx; u = 2x^2 + 1$

$\int_9^3 \frac{3}{u^2} du$

10) $\int_{-1}^0 \frac{4x}{(x^2 + 2)^2} dx; u = x^2 + 2$

$\int_3^2 \frac{2}{u^2} du$

11) $\int_1^2 \frac{8x}{(2x^2 + 2)^2} dx; u = 2x^2 + 2$

$\int_4^{10} \frac{2}{u^2} du$

Evaluate each definite integral.

$$12) \int_1^3 -\frac{6x}{(x^2 + 3)^2} dx; \quad u = x^2 + 3$$

$$-\frac{1}{2} = -0.5$$

$$13) \int_0^1 \frac{8x}{(4x^2 + 2)^2} dx; \quad u = 4x^2 + 2$$

$$\frac{1}{3} \approx 0.333$$

Evaluate each indefinite integral.

$$14) \int 12x^2 \sqrt{4x^3 + 5} dx$$

$$\frac{2}{3} (4x^3 + 5)^{\frac{3}{2}} + C$$

$$15) \int -3\sec^2 -3x \sqrt[3]{\tan -3x} dx$$

$$\frac{3}{4} \cdot (\tan -3x)^{\frac{4}{3}} + C$$

$$16) \int -\frac{5\sec -5xtan -5x}{\sec^5 -5x} dx$$

$$-\frac{1}{4\sec^4 -5x} + C$$

$$17) \int 2\csc -2x \cot -2x \sqrt{\csc -2x} dx$$

$$\frac{2}{3} \cdot (\csc -2x)^{\frac{3}{2}} + C$$

$$18) \int \frac{4e^{4x}}{(e^{4x} - 1)^3} dx$$

$$-\frac{1}{2(e^{4x} - 1)^2} + C$$

$$19) \int \frac{(-1 + \ln -4x)^5}{x} dx$$

$$\frac{1}{6} (-1 + \ln -4x)^6 + C$$

For each problem, find $F'(x)$.

$$20) F(x) = \int_{-3}^x (t+2) dt$$

$$F'(x) = x + 2$$

$$21) F(x) = \int_3^{3x} \frac{5}{(t-1)^3} dt$$

$$F'(x) = \frac{15}{(3x-1)^3}$$

$$22) F(x) = \int_0^{3x} (t^3 - t^2 - 1) dt$$

$$F'(x) = 81x^3 - 27x^2 - 3$$

$$23) F(x) = \int_x^{x^2} (t^3 - t^2 + 1) dt$$

$$F'(x) = 2x^7 - 2x^5 - x^3 + x^2 + 2x - 1$$

7) $f(x) = 2x^2 + 5$ $[0, 2]$ lower left sided
 $n=4$
rectangles

$$\frac{1}{2}(f(0) + f(\frac{1}{2}) + f(\frac{1}{2}) + f(1))$$
$$13.5 \text{ units}^2$$

8) $f(x) = 2x^2 + 2$ $[0, 1]$ lower
rect. left sided

$$\frac{1}{4}(f(0) + f(\frac{1}{4}) + f(\frac{1}{2}) + f(\frac{3}{4}))$$
$$2.4375 \text{ units}^2$$

24) $F(x) = \int_x^{2x} (t^3 - 3t^2 - 3) dt$

$$F'(x) = 15x^3 - 21x^2 - 3$$

For each problem, find the average value of the function over the given interval.

25) $f(x) = -x^3 + 2x^2 - 2; [0, 1]$

$$-\frac{19}{12} \approx -1.583$$

$$-\frac{1}{4} + \frac{2}{3} - 2$$

$$\frac{1}{1} \int_0^1 -x^3 + 2x^2 - 2 dx \\ = \left[-\frac{x^4}{4} + \frac{2x^3}{3} - 2x \right]_0^1$$

For each problem, find the average value of the function over the given interval. Then, find the values of c that satisfy the Mean Value Theorem for Integrals.

26) $f(x) = -2x^2 + 12x - 13; [4, 5]$

Average value of function: $\frac{1}{3} \approx 0.333$

Values that satisfy MVT: $\frac{9 + \sqrt{21}}{3} \approx 4.528$

For each problem, find the values of c that satisfy the Mean Value Theorem for Integrals.

27) $f(x) = 3(x-1)^{\frac{1}{2}}; [1, 4]$

$$\frac{7}{3} \approx 2.333$$

28) $f(x) = -\frac{5}{(x-2)^2}; [4, 6]$

$$2 + 2\sqrt{2} \approx 4.828$$

For each problem, find the average value of the function over the given interval.

29) $f(x) = -\frac{1}{2x-2}; [3, 4]$

$$\frac{-\ln 6 + \ln 4}{2} \approx -0.203$$

30) $f(x) = -\frac{4}{2x+2}; [1, 3]$

$$-\ln 8 + \ln 4 \approx -0.693$$

- 31) Let the velocity of a particle traveling along the x-axis be given by $v(t) = t^2 - 2t + 7$. Find the displacement and distance traveled by the particle from $t=2$ to $t=5$ seconds.

Distance trav. 39 units
 $S(2) = \frac{38}{3}$
 $S(5) = \frac{155}{3}$
 Displac $S(s) = 51\frac{2}{3}$ units

$$\begin{aligned} \int V(t) dt &= S(t) \\ \int_2^5 t^2 - 2t + 7 dt &= \left[\frac{t^3}{3} - \frac{2t^2}{2} + 7t \right]_2^5 = 39 \end{aligned}$$

$$\frac{b-a}{n}$$

5) $f(x) = 2x^2 + 4$ $[1, 2]$

$n=4$ Upper Sum
rectangles Right sided

$$\Delta x = \frac{1}{4}$$

$$= \frac{1}{4} (f(1\frac{1}{4}) + f(1\frac{1}{2}) + f(1\frac{3}{4}) + f(2))$$

$$9.4375 \text{ units}^2$$

6) $f(x) = x^2 + 6$ $[1, 2]$ $n=4$ rectangles

Upper Sum
Right sided

$$= \frac{1}{4} (f(1\frac{1}{4}) + f(1\frac{1}{2}) + f(1\frac{3}{4}) + f(2))$$

$$8.7188 \text{ units}^2$$