Session Notes

Suppose an object is moving along a straight line, such as the x-axis, so that its position x, as a function of time t, on that line is given by $y = x(t)$.

Average velocity of the object over the time interval t to $t + \Delta t$ is given by

$$\frac{x(t + \Delta t) - x(t)}{\Delta t}, \text{ or } \frac{\text{change in position}}{\text{change in time}}.$$

Instantaneous velocity of the object is the derivative of the position function $x(t)$ with respect to time. $v(t) = x'(t)$

Speed is the absolute value of the velocity. $Speed = |v(t)| = \left|\dfrac{dx}{dt}\right|$.

Acceleration is the derivative of velocity with respect to time. $a(t) = v'(t) = x''(t)$

$$\int v(t)\, dt = x(t) + c,$$

$$\int a(t)\, dt = v(t) + c$$

Total distance traveled from time $t = t_1$ to $t = t_2$ is given by

$$TDT = \int_{t_1}^{t_2} |v(t)|\, dt.$$

Speeding Up or Slowing Down

If the velocity and acceleration have the same sign (both positive or both negative), then speed is increasing. If an object's velocity is −40 miles per hour and the object accelerates −10 miles per hour per hour, the object is speeding up.

If the velocity and acceleration are opposite in sign (one is positive and the other is negative), then speed is decreasing. If an object's velocity is −40 miles per hour and the object accelerates 10 miles per hour per hour, the object is slowing down.

Sign Convention: When the object is moving in the right direction or moving upward then the velocity is positive (Graph of velocity vs. time is above the t axis). When the object is moving in the left direction or moving downward then the velocity is negative. A graph of velocity vs. time is shown below.
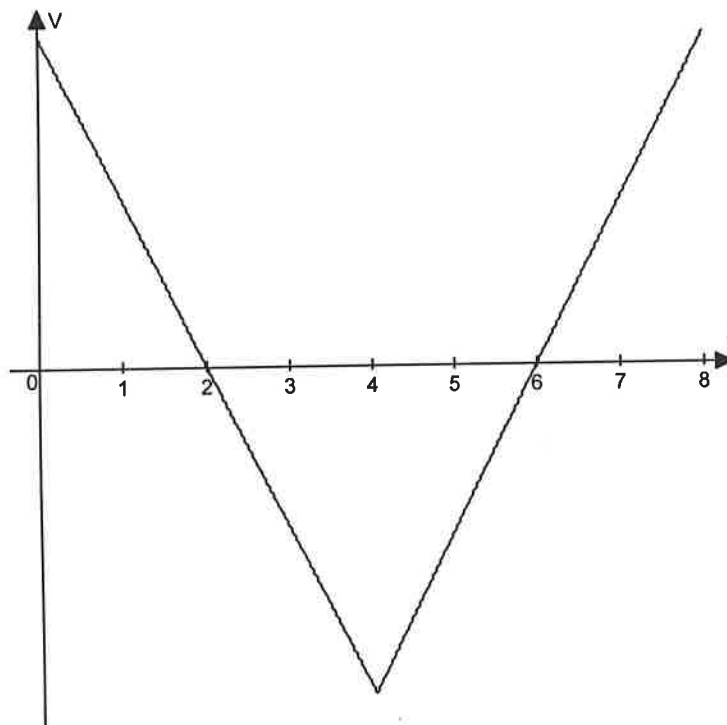
$0 < t < 2$	$v(+), a(-)$	Object is slowing down
$2 < t < 4$	$v(-), a(-)$	Object is speeding up
$4 < t < 6$	$v(-), a(+)$	Object is slowing down
$6 < t < 8$	$v(+), a(+)$	Object is speeding up

What you need to know about motion along the *x*-axis:

$$\frac{d}{dt} \; (slope) \downarrow \begin{array}{c} S(t) \\ V(t) \\ a(t) \end{array} \uparrow \int dt \;(Area)$$

When you see...	Think...
Initially	$t = 0$
At rest	$v(t) = 0$
At the origin	$x(t) = 0$
Velocity is positive	Particle is moving right (or up)
Velocity is negative	Particle is moving left (or down)
Average velocity (Given $x(t)$)	Change in position divided by change in time
Average velocity (Given $v(t)$)	$\dfrac{1}{b-a}\displaystyle\int_a^b v(t)\,dt$ (The average value of the velocity function.)
Instantaneous velocity	Velocity at an exact moment
Positive acceleration	Velocity is increasing
Negative acceleration	Velocity is decreasing
Speed	$\lvert v(t) \rvert$
Total Distance Traveled	$\displaystyle\int_a^b \lvert v(t) \rvert\, dt$
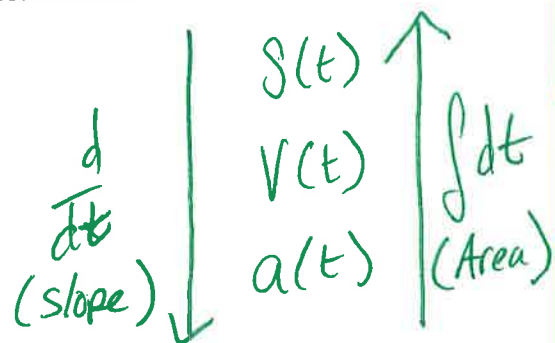
Displacement
(position shift) $\displaystyle\int_a^b v(t)\,dt$

$$\boxed{\tfrac{1}{2}a\,t^2 + V_0 t + S_0} \;\leftarrow\; a = gravity$$

Meter

$$\frac{ft}{}$$

$$S(t) = -\frac{32}{2}t^2 + V_0 t + S_0 \qquad\qquad S(t) = \frac{-9.8}{2}t^2 + V_0 t + S_0$$

$$V(t) = -32t + V_0 \qquad\qquad V(t) = -9.8t + V_0$$

$$a(t) = -32 \qquad\qquad a(t) = -9.8$$