

2.6 Related Rates Notes. (Rate of Change)

* Many things change with time. Our goal is to find the rate at which some quantity is changing by relating the quantity to other quantities whose rates of change are known.

ex: If x and y are both differentiable functions of t and are related by the equation $x^2 + y^2 = 25$.

Find $\frac{dy}{dt}$ when $x=3, y=4$ and $\frac{dx}{dt} = 8$

$$x^2 + y^2 = 25$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2y \frac{dy}{dt} = -2x \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-2x \frac{dx}{dt}}{2y}$$

$$\frac{dy}{dt} = \frac{-2(3)(8)}{2(4)}$$

$$\frac{dy}{dt} = -6$$

ex:



$$V = \frac{1}{3} \pi r^2 h$$

$$\text{Find } \frac{dV}{dt} = \underline{\hspace{2cm}}$$

$$\frac{dr}{dt} = 2 \text{ in/min}$$

$$h = 3r$$

$$r = 6 \text{ inches}$$

$$\frac{dV}{dt} = \cancel{\frac{1}{3} \pi} 3r^2 \frac{dr}{dt}$$

$$V = \frac{1}{3} \pi r^2 (3r)$$

$$V = \frac{1}{3} \pi 3r^3$$

$$\frac{dV}{dt} = \pi (3)(6)^2 (2) \quad \cancel{(\frac{1}{3} \pi 3r^3)}$$

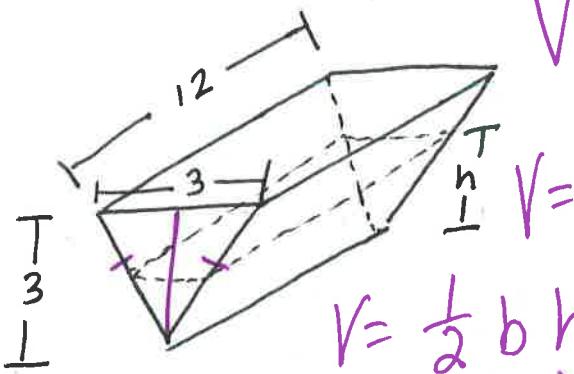
$$\frac{dV}{dt} = (\pi r^3) \frac{d}{dt}$$

$$\frac{dV}{dt} = 216\pi \frac{\text{in}^3}{\text{min}}$$

Ex: A water trough is 12 ft long and 3 ft across the top. Its ends are isosceles triangles with altitude 3 ft.

a) If water flows in at $2 \text{ ft}^3/\text{min}$. How fast is the water level rising when the depth "h" is 1 foot?

$$V = \frac{1}{2} b h l$$



$$V = \frac{1}{2} b h l$$

$$V = \frac{1}{2} b h (12)$$

$$V = \frac{1}{2} (\cancel{b})(\cancel{h})(h)(12)$$

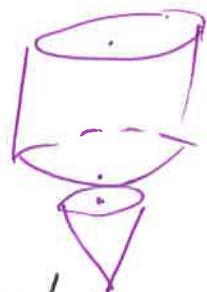
$$V = 6h^2$$

$$\frac{dV}{dt} = 6(2h) \frac{dh}{dt}$$

$$\begin{aligned} h &= 1 \text{ ft} \\ \frac{dV}{dt} &= 2 \text{ ft}^3/\text{min} \end{aligned}$$

$$\frac{dh}{dt} = ?$$

$$\begin{aligned} 2 &= 12(1) \frac{dh}{dt} \\ \frac{1}{6} &= \frac{dh}{dt} \end{aligned}$$



$$\frac{dh}{dt} = \frac{1}{6} \text{ ft/min}$$

b) If the water is rising at a rate of $\frac{3}{8} \text{ in/min}$ when $h=2$, determine the rate at which water is being pumped into the trough.

$$\frac{dV}{dt} = 12h \frac{dh}{dt}$$

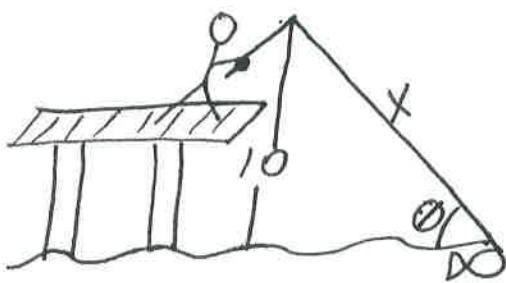
$$\frac{dV}{dt} = \frac{3}{8} \text{ in/min}$$

$$\frac{dV}{dt} = 18(\cancel{2})(\frac{3}{8})$$

$$\begin{aligned} h &= 2 \\ \frac{dV}{dt} &=? \end{aligned}$$

$$\frac{dV}{dt} = 9 \frac{\text{in}^3}{\text{min}}$$

ex) A fish is reeled in at a rate of 1 ft/sec. from a point 10 ft above water. At what rate is the angle θ between the line and the water changing when there is a total of 25 ft of line from the end of the rod and the water?



$$\frac{d\theta}{dt} = ?$$

$$x = 25$$

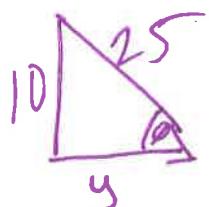
$$\frac{dx}{dt} = -1$$

$$\sin(\theta) = \frac{10}{x}$$

$$\sin(\theta) = 10x^{-1}$$

$$\cos(\theta) \frac{d\theta}{dt} = -10x^{-2} \frac{dx}{dt}$$

$$\rightarrow \frac{\sqrt{525}}{25} \frac{d\theta}{dt} = -10(25)^{-2}(-1)$$



$$y = \sqrt{25^2 - 10^2}$$

$$y = \sqrt{525}$$

$$\cos(\theta) = \frac{\sqrt{525}}{25}$$

$$\frac{\sqrt{525}}{25} \frac{d\theta}{dt} = \frac{2}{125}$$

$$\frac{d\theta}{dt} = 0.017 \text{ rad/sec}$$

Solving Related Rates Problems:

1. Always draw a picture
2. Find equation and identify the quantity you are looking for.
3. Use implicit differentiation with respect to time.
4. Plug and chug.

~~P.154 / 1-13 odd, 16, 17, 21, 25, 33~~

