

Calculus Ch 6

Test Review

Name: _____

Calculator Allowed!

1. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.
- (a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.
 - (b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.
 - (c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

2. The number of bacteria in a culture is increasing according to the law of exponential growth. After 5 hours there are 170 bacteria in the culture and after 10 hours there are 370 bacteria in the culture. Answer the following questions, rounding numerical answers to four decimal places.

- (i) Find the initial population.
- (ii) Write an exponential growth model for the bacteria population. Let t represent time in hours.
- (iii) Use the model to determine the number of bacteria after 20 hours.
- (iv) After how many hours will the bacteria count be 25,000?

FREE-RESPONSE QUESTION

A calculator may not be used on this question.

3. A differentiable function $f(x)$ is defined such that, for all values of x in its domain, $f(x) = 3 + \int_8^x f(\sqrt[3]{t}) dt$.
- What is the domain of $f(x)$?
 - For what value(s) of x is $f(x) = 3$?
 - Show that $f'(x) = 3x^2 f(x)$.
 - Solve the differential equation in (c) to find $f(x)$ in terms of x only.

Graphing Calculator Allowed

1. Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = x^5 e^{x^6}$$

a) $y = \frac{x^6}{6} e^{x^6} + C$

b) $y = 6e^{x^6} + C$

c) $y = \frac{1}{5} e^{x^6} + C$

d) $y = 5e^{x^6} + C$

e) $y = \frac{1}{6} e^{x^6} + C$

2. Which of the following is a solution of the differential equation $xy' - 3y = x^4 e^x$?

a) $y = x^3 e^x$

b) $y = 5e^{2x} - 6\sin 2x$

c) $y = 4e^{-2x}$

d) $y = \ln x$

e) $y = 3x^4 e^{2x}$

3. Which of the following is a solution of the differential equation $y^{(4)} - 256y = 0$?

a) $y = x^2(4 + e^x)$

b) $y = 3\ln x$

c) $y = e^{-4x}$

d) $y = -4x \cos x$

e) $y = 2e^{-4x} + x \sin x$

4. Find the particular solution of the differential equation $\frac{dr}{ds} = e^{-2s}$ that satisfies the initial condition $r(0) = 0$.

a) $r = \ln(2 + e^{-2s}) + C$

b) $r = \ln(2) - \ln(1 + e^{-2s})$

c) $r = \ln\left(\frac{3 + e^{-2s}}{2}\right)$

d) $r = e^{-2s}$

e) $r = (1 + e^{-2s})^2$

5. Find an equation of the graph that passes through the point (7,9) and has the slope $y' = \frac{8y}{9x}$.

a) $y = 9\left(\frac{x}{7}\right)$

b) $y = 9(7x)^{\frac{1}{9}}$

c) $y = 7\left(\frac{8x}{9}\right)^9$

d) $y = xe^{\frac{8.7}{9}}$

e) $y = \frac{9}{8x} - \ln(7x) + 9$

6. Write and solve the differential equation that models the following verbal statement. Evaluate the solution at the specified value of the independent variable, rounding your answer to four decimal places:

The rate of change of y is proportional to y . When $x = 0$, $y = 56$ and when $x = 2$, $y = 72$. What is the value of y when $x = 10$?

a) $y(10) = 196.7480$

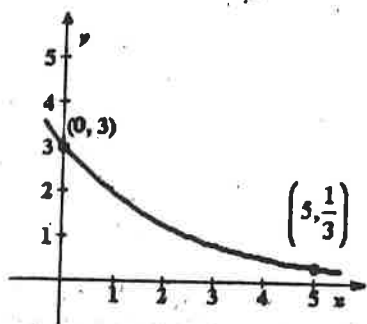
b) $y(10) = 200.4980$

c) $y(10) = 202.9580$

d) $y(10) = 192.9980$

e) $y(10) = 187.8280$

7. Find the exponential function $y = Ce^{kx}$ that passes through the two given points. Round your values of C and k to four decimal places.



a) $y = 3e^{-1.8067x}$

b) $y = 3e^{0.3563x}$

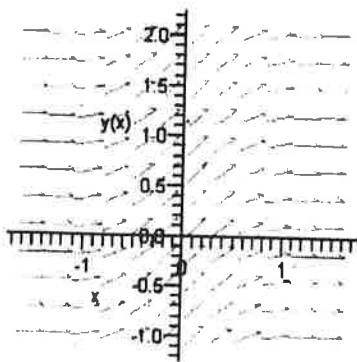
c) $y = 3e^{0.2239x}$

d) $y = 3e^{-0.9027x}$

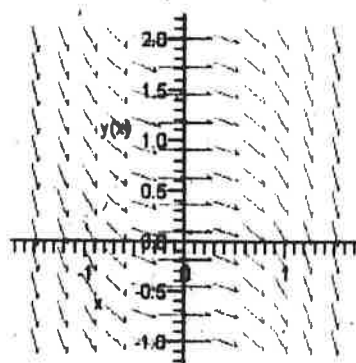
e) $y = 3e^{-0.4394x}$

8. Identify the slope field of the differential equation $\frac{dy}{dx} = e^{-x^2}$.

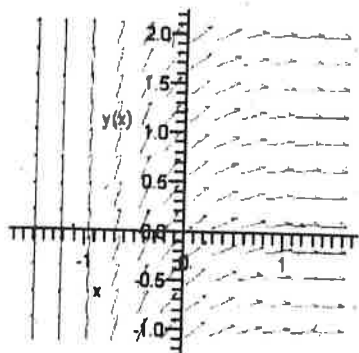
a)



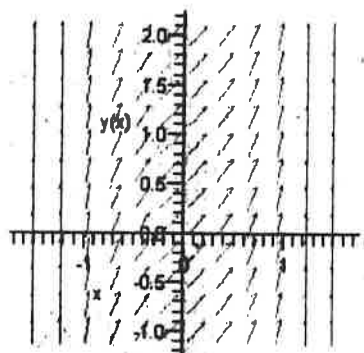
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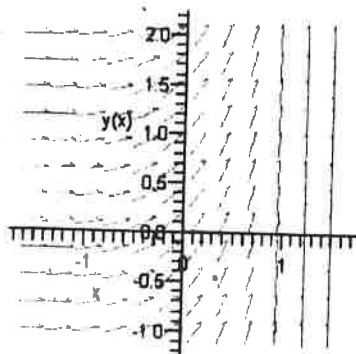
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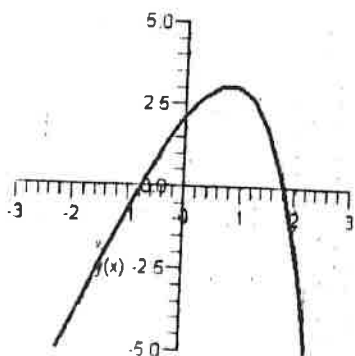


c)

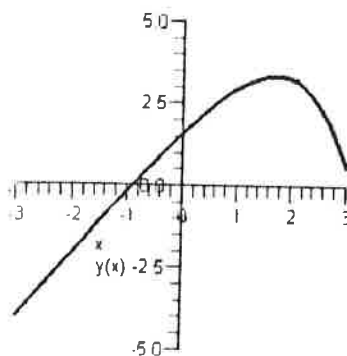


9. Sketch the slope field for the differential equation $y' = y - 4x$ and use the slope field to sketch the solution that passes through the point $(1, 3)$.

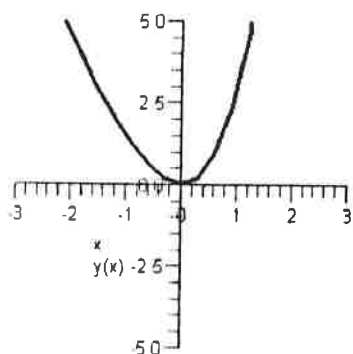
a)



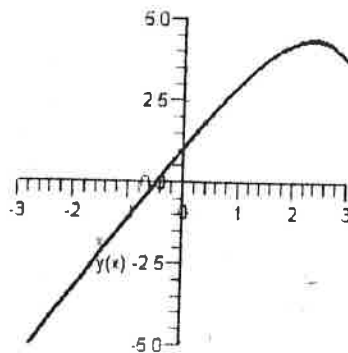
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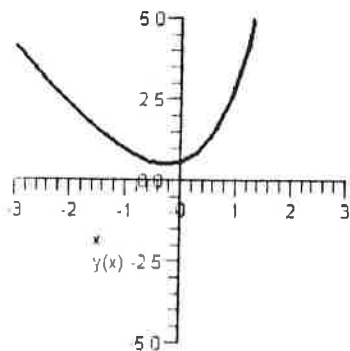
b)



e)



c)



10. The rate of change of N is proportional to N . When $t = 0$, $N = 180$ and when $t = 1$, $N = 480$. What is the value of N when $t = 2$? Round your answer to three decimal places.

- a) 1,230.000
- b) 1,280.000
- c) 59.112
- d) 1,310.000
- e) 3,840.000

11. The half-life of the carbon isotope C-14 is approximately 5715 years. If the initial quantity of the isotope is 40 g, what is the amount left after 5000 years? Round your answer to two decimal places.
- a) 22.31 g
 - b) 17.00 g
 - c) 21.81 g
 - d) 29.54 g
 - e) 10.91 g
12. The isotope ^{239}Pu has a half-life of 24,100 years. After 2,000 years, a sample of the isotope is reduced to 1.1 grams. What was the initial size of the sample (in grams)? How much will remain after 20,000 years (i.e., after another 18,000 years)? Round your answers to four decimal places.
- a) 0.8156, 0.4588
 - b) 1.5147, 0.8521
 - c) 1.8642, 1.0488
 - d) 1.1651, 0.6555
 - e) 1.6312, 0.9177
13. The initial investment in a savings account in which interest is compounded continuously is \$973. If the time required to double the amount is $7\frac{3}{4}$ years, what is the amount after 14 years? Round your answer to the nearest cent.
- a) \$3403.37
 - b) \$3892.00
 - c) \$10,879.00
 - d) \$3515.35
 - e) \$3003.37
14. A container of hot liquid is placed in a freezer that is kept at a constant temperature of 20°F . The initial temperature of the liquid is 180°F . After 5 minutes, the liquid's temperature is 68°F . How much longer will it take for its temperature to decrease to 34°F ? Round your answer to two decimal places.
- a) 3.07 minutes
 - b) 5.12 minutes
 - c) 4.61 minutes
 - d) 5.63 minutes
 - e) 2.05 minutes

15. A calf that weighs 60 pounds at birth gains weight at the rate $\frac{dw}{dt} = k(1200 - w)$, where w is weight in pounds and t is time in years. If the animal is sold when its weight reaches 700 pounds, find the time of sale using the model $w = 1200 - 1140e^{-1.2t}$. Round your answer to two decimal places.

- a) 0.69 year
- b) 5.03 years
- c) 2.05 years
- d) 0.49 year
- e) 1.47 years

16. A calf that weighs 55 pounds at birth gains weight at the rate $\frac{dw}{dt} = k(1100 - w)$, where w is weight in pounds and t is time in years. What is the maximum weight of the animal if one uses the model $w = 1100 - 1045e^{-1.1t}$?

- a) 1045 lb
- b) 655 lb
- c) 1100 lb
- d) 1700 lb
- e) 1155 lb

17. Solve the homogeneous differential equation $y' = \frac{9x + 11y}{x}$.

- a) $y = \frac{9}{11}x + C \cdot x^9$
- b) $y = -\frac{11}{8}x - x^{22} + C$
- c) $y = -\frac{9}{10}x + C \cdot x^{11}$
- d) $y = \frac{9}{10}x^{11} + C$
- e) $y = \ln(9x + 11) + C$

Graphing Calculator Is Not Allowed!

1. Solve the differential equation $y' = \frac{\sqrt{x}}{2y}$.

a) $y = \sqrt{\frac{2}{3}x^{3/2} + C}$

b) $2y^2 = 4x^{3/2} + C$

c) $6y^2 = 2x^{3/2} + C$

d) $6y^2 = 4x^{3/2} + C$

e) $4y^2 = 6x^{3/2} + C$

2. Solve the differential equation.

$$y' = x(-5 + y)$$

a) $\ln|-5 + y| = x^2 + C$

b) $\ln|-5 + y| = x + C$

c) $3\ln|-5 + y| = x^3 + C$

d) $4\ln|-5 + y| = x^4 + C$

e) $2\ln|-5 + y| = x^2 + C$

3. Solve the differential equation.

$$y' = \frac{y}{x}$$

a) $2\ln y = -x^3 + C$

b) $y^2 = x^3 + C$

c) $2\ln y = x^2 + C$

d) $y^2 = x^2 + C$

e) $y^2 = -x^2 + C$

4. Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = -2x^3 - 9x$$

a) $2x^4 - 9x^2 + C$

b) $6x^2 - 9 + C$

c) $-\frac{1}{2}x^4 - \frac{9}{2}x^2 + C$

d) $-2x^3 - 9 + C$

e) $-\frac{1}{2}x^4 - \frac{9}{2}x^2 + x + C$

5. Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = -8x^2 - 7x$$

- a) $y = -8x^2 - 7 + C$
- b) $y = -8x^3 - 7x^2 + C$
- c) $y = -16x - 7 + C$
- d) $y = -\frac{8}{3}x^3 - \frac{7}{2}x^2 + C$
- e) $y = -8x^2 - 7x + C$

6. Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = x\sqrt{-x+5}$$

- a) $y = \frac{2}{5}(-x+5)^{5/2} - \frac{10}{3}(-x+5)^{3/2} + C$
- b) $y = \frac{2}{5}(-x+5)^{3/2} - \frac{10}{3}(-x+5)^{5/2} + C$
- c) $y = \frac{2}{5}(-x+5)^2 - \frac{10}{3}(-x+5) + C$
- d) $y = (-x+5)^{3/2} - (-x+5)^{5/2} + C$
- e) $y = \frac{2}{5}(-x+5)^{3/2} + \frac{10}{3}(-x+5)^{5/2} + C$

7. Use integration to find a general solution of the differential equation $\frac{dy}{dx} = 5x \cos(15x^2)$.

- a) $\frac{\sin(30x^2)}{3} + C$
- b) $\frac{\sin(15x^2)}{6} + C$
- c) $\frac{\cos(15x^2)}{6x} + C$
- d) $\frac{2x \sin(30x^2)}{3x} + C$
- e) $\frac{x \cos(15x^2)}{6} + C$

Write and solve the differential equation that models the following verbal statement:

8. The rate of change of Q with respect to x is proportional to $40 - x$.

- a) $\frac{dQ}{dx} = k(40 - x)^{-1}$, $Q = -k \ln(40 - x) + C$
- b) $\frac{dQ}{dx} = k(40 - x)$, $Q = -\frac{k}{2}(40 - x)^2 + C$
- c) $\frac{dQ}{dx} = k(40 - x)^2$, $Q = -\frac{k}{3}(40 - x)^3 + C$
- d) $\frac{dQ}{dx} = k(40 - x)^3$, $Q = -\frac{k}{4}(40 - x)^4 + C$
- e) $\frac{dQ}{dx} = k(40 - x)^{-1}$, $Q = -k \ln(40 - x)^2 + C$

9. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{9x^2}{10y^2}$.

- a) $y = \frac{9}{10}x^3 + C$
- b) $y = \sqrt{\frac{10}{9}x^3 + C}$
- c) $y = \sqrt{9x^3 + 10C}$
- d) $y = \sqrt{\frac{9}{10}x^3 + C}$
- e) $y = \sqrt{\frac{x^3}{10} + C}$

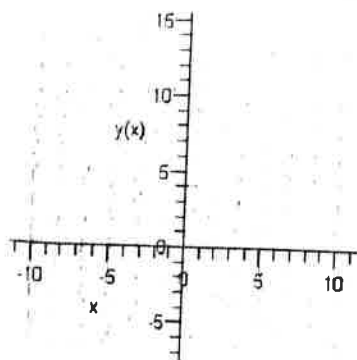
10. Find the particular solution of the differential equation $19x + 4yy' = 0$ that satisfies the initial condition $y = 8$ when $x = 3$, where $19x^2 + 4y^2 = C$ is the general solution.

- a) $19x^2 + 4y^2 = 1252$
- b) $19x^2 + 4y^2 = 427$
- c) $19x^2 + 4y^2 = 235$
- d) $19x^2 + 4y^2 = 203$
- e) $19x^2 + 4y^2 = 265$

11. Find the function $y = f(t)$ passing through the point $(0, 18)$ with the first derivative $\frac{dy}{dt} = \frac{2}{3}y$.

- a) $y(t) = 18e^{\frac{2}{3}t^2}$
- b) $y(t) = e^{\frac{2}{3}t^2} + 18$
- c) $y(t) = \frac{2}{3}t^2 + 18$
- d) $y(t) = 18e^{\frac{2}{3}t}$
- e) $y(t) = e^{\frac{2}{3}t} + 18$

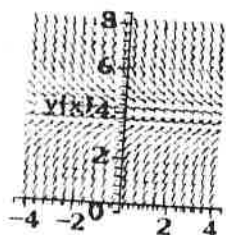
12. Use the differential equation $\frac{dy}{dx} = \frac{3x}{y}$ and its slope field to find the slope at the point $(-5, 5)$.



- a) -75
- b) -25
- c) 15
- d) -15
- e) -3

13. Sketch a few solutions of the differential equation on the slope field and then find the general solution analytically.

$$\frac{dy}{dx} = 4 - y$$



- a) $y = C \ln(4 - y)$
- b) $y = 4 + Ce^{-x}$
- c) $y = 4 - 2Ce^x$
- d) $y = C \ln(y - 4)$
- e) $y = 4x + Ce^{-x}$

