

4.1-4.4 Review Key Calculus

$$1) \frac{2x^{3/2}}{3} + x^{1/2} + C$$

$$2) t + \csc t + C$$

$$3) \overset{Rt}{187/16} \approx 11.6875$$

$$\overset{Lt}{155/16} \approx 9.6875$$

$$4) \frac{n^2(2n+3)}{6} \text{ or } \frac{2n^3+3n^2}{6}$$

$$5) 15$$

$$6) \int_0^3 \sqrt{x^2+4} \, dx$$

$$7) -10/3$$

$$8) 57.1125$$

$$9) 2\sqrt{3}/3$$

$$10) \pi/4$$

$$11) C \approx 0.438, 1.7908$$

$$12) x = \pm \sqrt{3}$$

$$13) F(2) = -6$$

$$F(5) = 15$$

$$F(8) = 72$$

$$14) F'(x) = \sqrt[4]{x}$$

$$15) F'(x) = 0$$

4.1-4.4 Review

Name: Key

Find the indefinite integral and check the results by differentiation

$$1) \int (\sqrt{x} + \frac{1}{2\sqrt{x}}) dx$$

$$= \int x^{1/2} + \frac{1}{2} x^{-1/2} dx$$

$$= \frac{2x^{3/2}}{3} + x^{1/2} + C$$

$$2) \int (1 - \csc(t)\cot(t)) dt$$

$$= t + \csc t + C$$

Use left and right endpoints and the given number of rectangles to find two approx. for area.

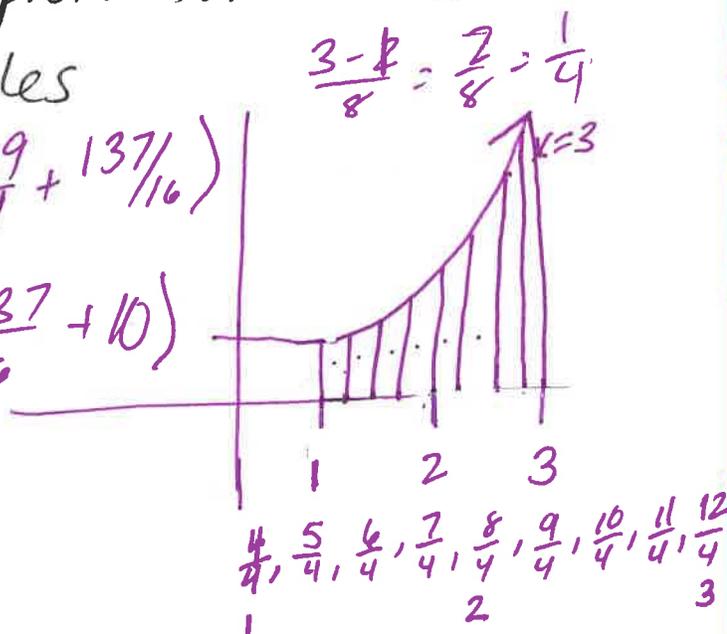
3) $f(x) = x^2 + 1$ $[1, 3]$, 8 rectangles

$$L = \frac{1}{4} (2 + \frac{41}{16} + \frac{13}{4} + \frac{65}{16} + 5 + \frac{97}{16} + \frac{29}{4} + \frac{137}{16})$$

$$= \frac{155}{16} = 9.6875$$

$$R = \frac{1}{4} (\frac{41}{16} + \frac{13}{4} + \frac{65}{16} + 5 + \frac{97}{16} + \frac{29}{4} + \frac{137}{16} + 10)$$

$$= \frac{187}{16} = 11.6875$$



Evaluate the sum

$$1) \sum_{i=1}^n (i^2 - 1)$$

$$\frac{2n^2 + 3n + 1}{6} - \frac{6(n)}{6} = \frac{2n^3 + 3n^2 + n - 6n}{6} = \frac{2n^3 + 3n^2 - 5n}{6}$$

$$= \frac{n^2(2n+3)}{6}$$

Evaluate the definite integral by the limit definition

$$5) \int_{-2}^1 (2x^2+3) dx \quad \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x \quad \Delta x = \frac{1-(-2)}{n} = \frac{3}{n}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n (2(-2 + \frac{3}{n}i)^2 + 3) (\frac{3}{n}) = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{33}{n} - \frac{72}{n^2}i + \frac{54}{n^2}i^2$$

$$= \lim_{n \rightarrow \infty} \left(\frac{33n}{n} - \frac{72}{n^2} \left(\frac{n(n+1)}{2} \right) + \frac{54}{n^3} \left(\frac{n(n+1)(2n+1)}{6} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \left(33 - \frac{72n^2 + 72n}{2n^2} + \frac{90n^3}{n^3} \dots \right) = 33 - \frac{72}{2} + \frac{108}{6} = \boxed{15}$$

Write the limit as a definite integral

$$6) \lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{c_i^2 + 4} \Delta x_i \quad [0, 3]$$

$$\int_0^3 \sqrt{x^2 + 4} dx$$

Evaluate the definite Integral.

$$7) \int_{-1}^1 (t^2 - 2) dt$$

$$\left. \frac{t^3}{3} - 2t \right|_{-1}^1 = \left(\frac{1}{3} - 2 \right) - \left(-\frac{1}{3} + 2 \right)$$

$$= -\frac{10}{3}$$

$$8) \int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}} dx$$

$$= \int_{-8}^{-1} \frac{x}{2x^{1/3}} - \frac{x^2}{2x^{1/3}} dx = \int_{-8}^{-1} \frac{x^{2/3}}{2} - \frac{x^{5/3}}{2} dx$$

$$= \left[\frac{3}{10} x^{5/3} - \frac{3}{2(8)} x^{8/3} \right]_{-8}^{-1} = \frac{4569}{80} = 57.1125$$

$$9) \int_{-\pi/6}^{\pi/6} \sec^2 x dx$$

$$= \left[\tan x \right]_{-\pi/6}^{\pi/6} = \frac{\sqrt{3}}{3} - \left(-\frac{\sqrt{3}}{3} \right)$$

$$= \frac{2\sqrt{3}}{3}$$

$$10) \int_0^{\pi/4} \frac{\sec^2 \theta}{\tan^2 \theta + 1} d\theta$$

$$= \int_0^{\pi/4} \frac{\sec^2 \theta}{\sec^2 \theta} d\theta = \int_0^{\pi/4} 1 d\theta$$

$$= \pi/4$$

find the values of c guaranteed by the Mean Value theorem for Integrals.

$$\int_0^2 (x-2\sqrt{x}) dx = 2 - \frac{8\sqrt{2}}{3}$$

11) $f(x) = x - 2\sqrt{x}, [0, 2]$

$$f(c)(2-0) = \frac{6-8\sqrt{2}}{3}$$

$$c-2\sqrt{c} = \frac{3-4\sqrt{2}}{3}$$

$$c-2\sqrt{c}+1 = \frac{3-4\sqrt{2}+1}{3}$$

$$(\sqrt{c}-1)^2 = \frac{6-4\sqrt{2}}{3}$$

$$\sqrt{c}-1 = \pm \sqrt{\frac{6-4\sqrt{2}}{3}}$$

$$c = \left[1 \pm \sqrt{\frac{6-4\sqrt{2}}{3}} \right]^2$$

$C \approx 0.438$ or $C \approx 1.7908$

find the average value of the function over the given interval, for which the function equals its average value and all values x

12) $f(x) = 9 - x^2, [-3, 3]$

$$\frac{1}{3-(-3)} \int_{-3}^3 (9-x^2) dx = \frac{1}{6} [9x - \frac{1}{3}x^3]_{-3}^3 = 6$$

$$9 - x^2 = 6 \quad x = \pm\sqrt{3} \approx \pm 1.7321$$

find F as a function of x and evaluate it at $x=2, x=5, \text{ and } x=8$

13) $F(x) = \int_0^x (4t-7) dt = [2t^2-7t]_0^x = 2x^2-7x$

$$F(2) = 2(2^2) - 7(2) = -6$$

$$F(5) = 2(5^2) - 7(5) = 15$$

$$F(8) = 2(8^2) - 7(8) = 72$$

Use the second fundamental theorem of calculus.

$$14) F(x) = \int_1^x \sqrt[4]{t} dt$$

$$F' = \sqrt[4]{x}$$

Find $F'(x)$

$$15) F(x) = \int_{-x}^x t^3 dt = \left[\frac{t^4}{4} \right]_{-x}^x = 0$$
$$= \frac{x^4}{4} - \frac{(-x)^4}{4} = 0$$

$$F'(x) = 0$$

16) Integrate by Sub.

$$\int (1+2x)^4 (2) dx$$
$$= \int u^4 du \quad \begin{array}{l} u = 1+2x \\ du = 2dx \end{array}$$

$$= \frac{u^5}{5} + C$$

$$= \frac{(1+2x)^5}{5} + C \quad \text{OR} \quad \frac{1}{5} (1+2x)^5 + C$$

$$\begin{aligned}
 17) \int \frac{t+2t^2}{\sqrt{t}} dt &= \int \frac{t}{\sqrt{t}} + \frac{2t^2}{\sqrt{t}} dt = \int \frac{t}{t^{1/2}} + \frac{2t^2}{t^{1/2}} dt \\
 &= \int t^{1/2} + 2t^{3/2} dt = \frac{2t^{3/2}}{3} + \frac{2(2)t^{5/2}}{5} + C \\
 &= \boxed{\frac{2}{3}t^{3/2} + \frac{4}{5}t^{5/2} + C}
 \end{aligned}$$

$$\begin{aligned}
 18) \frac{1}{2} \int \frac{x}{\sqrt{2x-1}} 2 dx \quad & u = 2x-1 \quad \frac{u+1}{2} = x \\
 & du = 2 dx \\
 & = \frac{1}{2} \int \frac{x}{\sqrt{u}} du = \frac{1}{2} \int \frac{\frac{u}{2} + \frac{1}{2}}{\sqrt{u}} du = \frac{1}{2} \int \frac{1}{\sqrt{u}} \left(\frac{u}{2} + \frac{1}{2} \right) du \\
 & = \frac{1}{2} \int \frac{u}{2\sqrt{u}} + \frac{1}{2\sqrt{u}} du = \frac{1}{2} \int \frac{1}{2} u^{1/2} + \frac{1}{2} u^{-1/2} du = \frac{1}{4} \int u^{1/2} + u^{-1/2} du \\
 & = \frac{1}{4} \left(\frac{2u^{3/2}}{3} + \frac{2u^{1/2}}{1} \right) + C = \frac{2}{12} (2x-1)^{3/2} + \frac{2}{4} (2x-1)^{1/2} + C \\
 & = \boxed{\frac{1}{6} (2x-1)^{3/2} + \frac{1}{2} (2x-1)^{1/2} + C}
 \end{aligned}$$

$$\begin{aligned}
 19) \int_{-2}^0 (3^3 - 5^2) dx &= \int_{-2}^0 27 - 25 dx = \int_{-2}^0 2 dx = 2x \Big|_{-2}^0 = 2(0) - 2(-2) = \boxed{4}
 \end{aligned}$$

$$\begin{aligned}
 20) \int \sqrt{\tan x} \cdot \sec^2 x dx \quad & u = \tan x \\
 & du = \sec^2 x dx \\
 \int \sqrt{u} du &= \int u^{1/2} du = \frac{2u^{3/2}}{3} + C = \boxed{\frac{2}{3} (\tan x)^{3/2} + C}
 \end{aligned}$$

