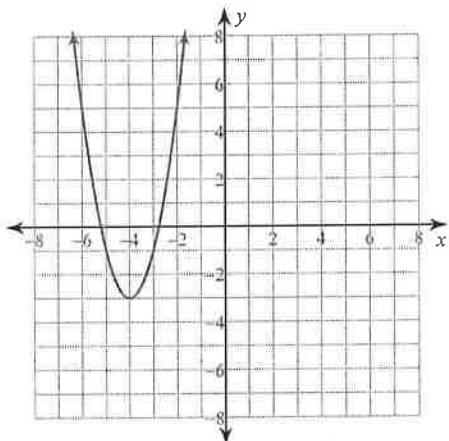


Unit Review for Applications of Differentiation

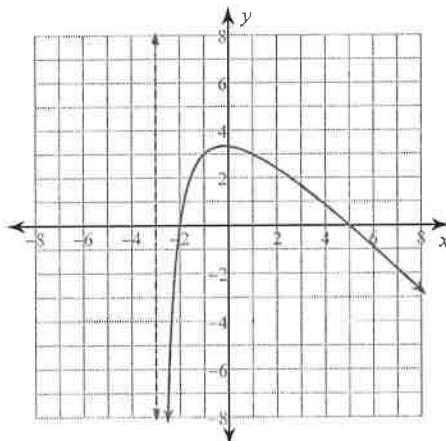
For each problem, determine if Rolle's Theorem can be applied. If it can, find all values of c that satisfy the theorem. If it cannot, explain why not. Then use the provided graph to sketch the function, including the secant and tangent lines.

1) $y = 2x^2 + 16x + 29$; $[-5, -3]$



$\{-4\}$

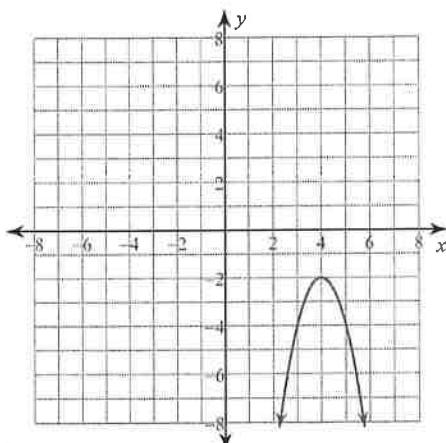
2) $y = \frac{-x^2 + 3x + 10}{x + 3}$; $[-2, 5]$



$\{-3 + 2\sqrt{2}\}$

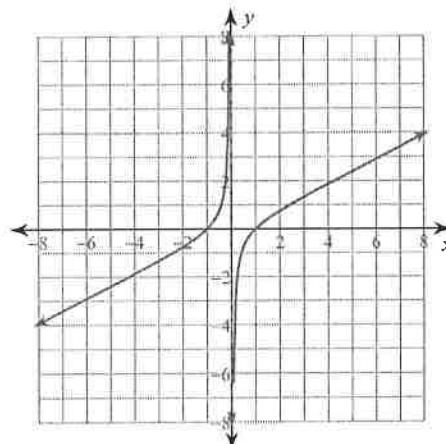
For each problem, find the values of c that satisfy the Mean Value Theorem. You may use the provided graph to sketch the function.

3) $y = -2x^2 + 16x - 34$; $[3, 5]$



$\{4\}$

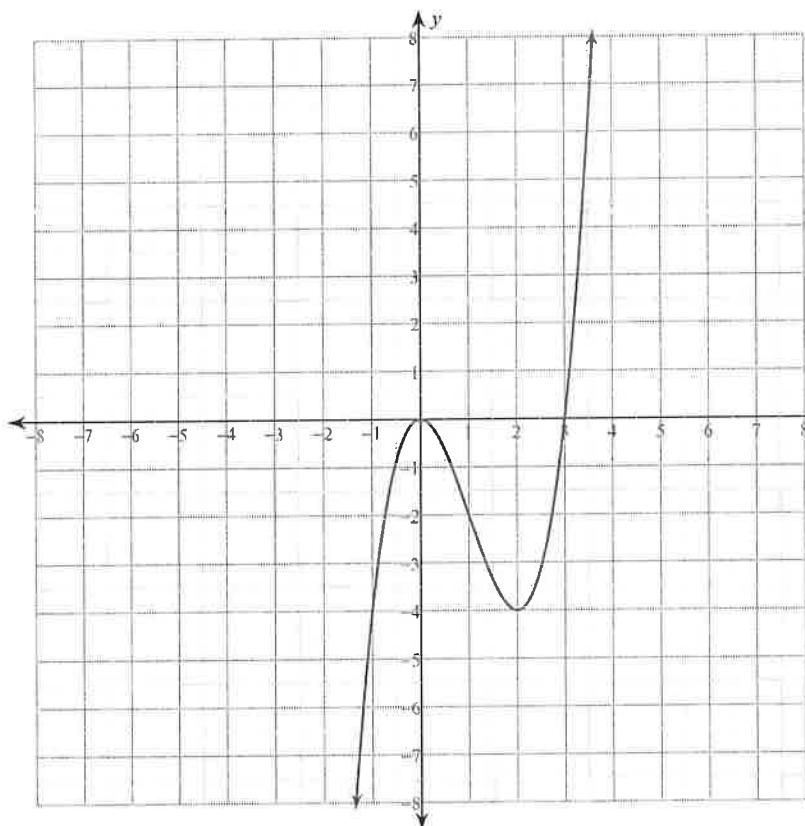
4) $y = \frac{x^2 - 1}{2x}$; $[1, 6]$



$\{\sqrt{6}\}$

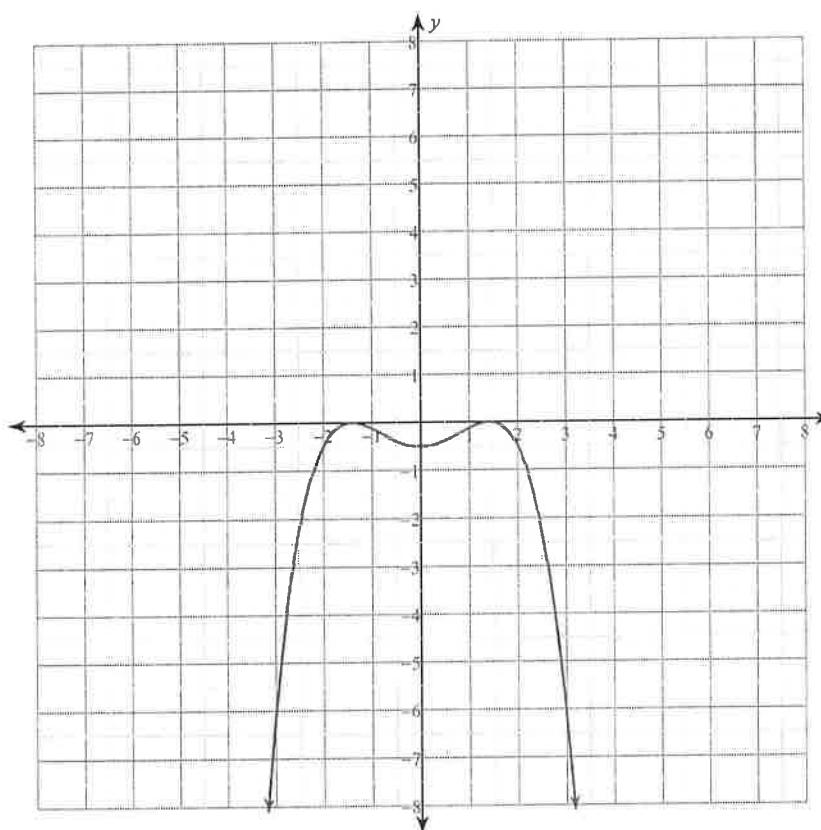
For each problem, find the: x and y intercepts, asymptotes, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, relative minima and maxima, and domain and Range. Using this information, sketch the graph of the function.

5) $y = x^3 - 3x^2$



- x-intercepts at $x = 0, 3$
- y-intercept at $y = 0$
- No vertical asymptotes exist.
- No horizontal asymptotes exist.
- Critical points at: $x = 0, 2$
- Increasing: $(-\infty, 0), (2, \infty)$
- Decreasing: $(0, 2)$
- Inflection point at: $x = 1$
- Concave up: $(1, \infty)$
- Concave down: $(-\infty, 1)$
- Relative minimum: $(2, -4)$
- Relative maximum: $(0, 0)$

$$6) \quad y = -\frac{x^4}{8} + \frac{x^2}{2} - \frac{1}{2}$$



x -intercepts at $x = -\sqrt{2}, \sqrt{2}$

y -intercept at $y = -\frac{1}{2}$

No vertical asymptotes exist.

No horizontal asymptotes exist.

Critical points at: $x = -\sqrt{2}, 0, \sqrt{2}$

Increasing: $(-\infty, -\sqrt{2}), (0, \sqrt{2})$

Decreasing: $(-\sqrt{2}, 0), (\sqrt{2}, \infty)$

Inflection points at: $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$

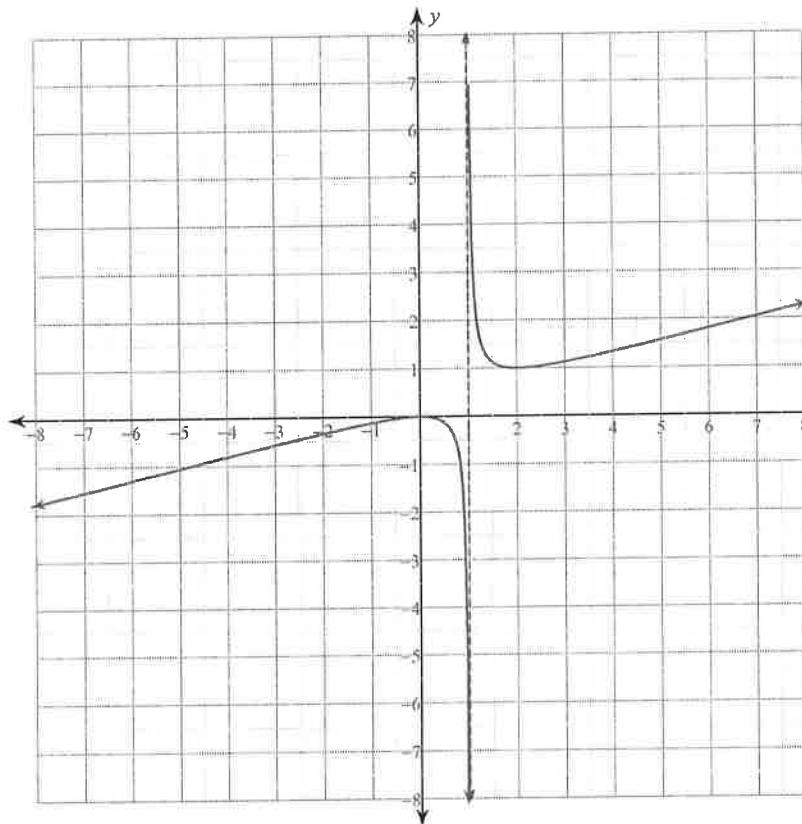
Concave up: $\left(-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right)$

Concave down: $\left(-\infty, -\frac{\sqrt{6}}{3}\right), \left(\frac{\sqrt{6}}{3}, \infty\right)$

Relative minimum: $\left(0, -\frac{1}{2}\right)$

Relative maxima: $(-\sqrt{2}, 0), (\sqrt{2}, 0)$

7) $y = \frac{x^2}{4x - 4}$



x -intercept at $x = 0$

y -intercept at $y = 0$

Vertical asymptote at: $x = 1$

No horizontal asymptotes exist.

Slant asymptote: $y = \frac{x}{4} + \frac{1}{4}$

Critical points at: $x = 0, 2$

Increasing: $(-\infty, 0), (2, \infty)$

Decreasing: $(0, 1), (1, 2)$

No inflection points exist.

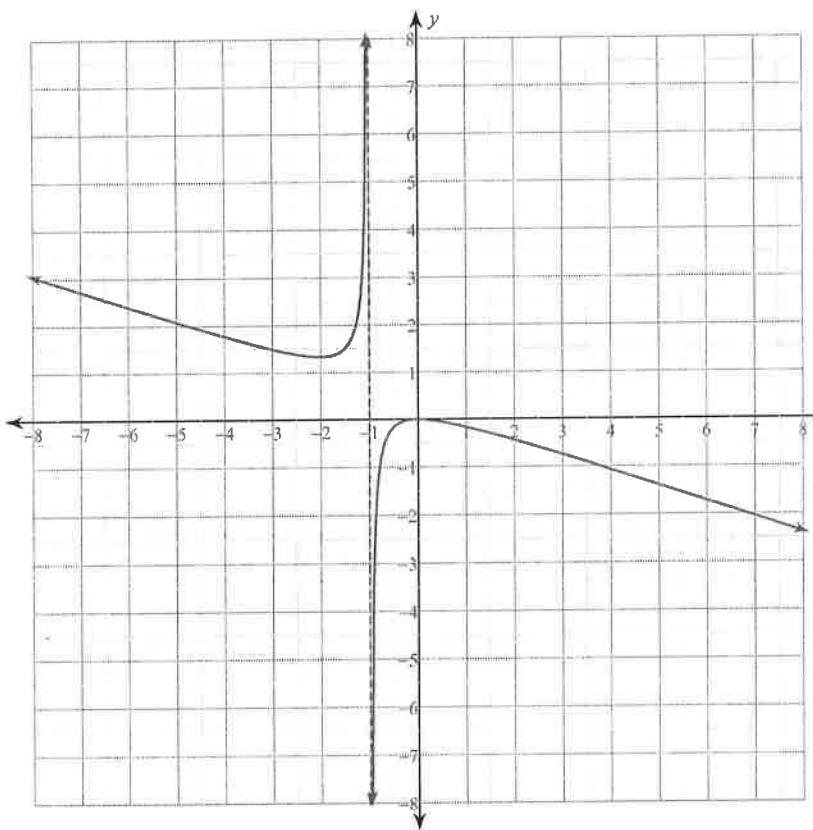
Concave up: $(1, \infty)$

Concave down: $(-\infty, 1)$

Relative minimum: $(2, 1)$

Relative maximum: $(0, 0)$

8) $y = -\frac{x^2}{3x + 3}$



x -intercept at $x = 0$

y -intercept at $y = 0$

Vertical asymptote at: $x = -1$

No horizontal asymptotes exist.

Slant asymptote: $y = -\frac{x}{3} + \frac{1}{3}$

Critical points at: $x = -2, 0$

Increasing: $(-2, -1), (-1, 0)$

Decreasing: $(-\infty, -2), (0, \infty)$

No inflection points exist.

Concave up: $(-\infty, -1)$

Concave down: $(-1, \infty)$

Relative minimum: $\left(-2, \frac{4}{3}\right)$

Relative maximum: $(0, 0)$

For each problem, find the differential dy .

9) $y = \sqrt{x}$

$$dy = \frac{1}{2x^{\frac{1}{2}}} dx$$

10) $y = -x^3 + 2$

$$dy = -3x^2 dx$$

11) $y = -x^2 + 2x + 1$

$$dy = (-2x + 2)dx$$

12) $y = x^3 + 3$

$$dy = 3x^2 dx$$

For each problem, find the general formulas for dy and Δy .

13) $y = -\sqrt{x}$

$$dy = -\frac{1}{2x^{\frac{1}{2}}} dx$$

$$\Delta y = -\sqrt{x + \Delta x} + \sqrt{x}$$

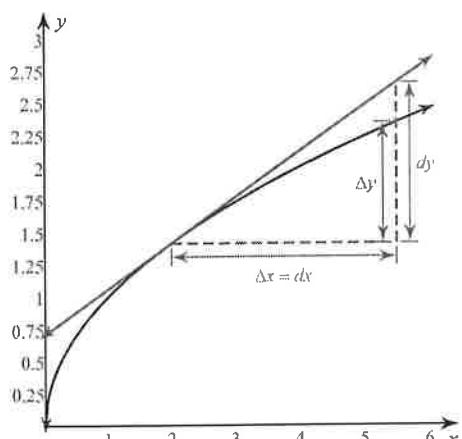
14) $y = \sqrt{x}$

$$dy = \frac{1}{2x^{\frac{1}{2}}} dx$$

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$

For each problem, find dy and Δy , given x_0 and $dx = \Delta x$. You may use the provided graph of the function to sketch dx , Δx , dy , and Δy .

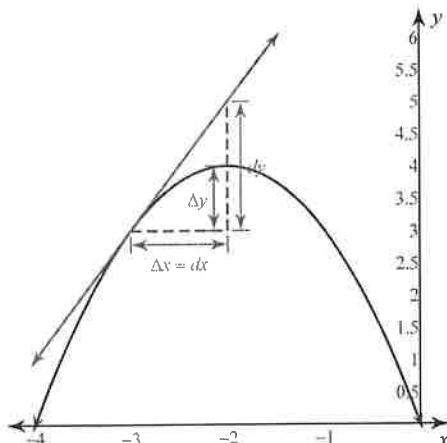
15) $y = \sqrt{x}; x_0 = 2, dx = \Delta x = \frac{7}{2}$



$$dy = \frac{7\sqrt{2}}{8} \approx 1.2374$$

$$\Delta y = \frac{-2\sqrt{2} + \sqrt{22}}{2} \approx 0.931$$

16) $y = -x^2 - 4x; x_0 = -3, dx = \Delta x = 1$



$$dy = 2$$

$$\Delta y = 1$$

Use differentials to solve each problem.

- 17) The radius of a circle is measured to be 6 ft, with a possible error of $\pm \frac{1}{10}$ ft. Estimate the possible propagated error in the calculated area.

$$\pm \frac{6\pi}{5} \approx \pm 3.7699 \text{ ft}^2$$

- 18) The sides of a square are measured to be 5 ft, with a possible error of $\pm \frac{1}{5}$ ft. Estimate the possible propagated error in the calculated area.

$$\pm 2 \text{ ft}^2$$