Calculus Ch 6
Test Review

Name: Key

Calculator Allowed!

1. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

(a) $f'(1) = \left.\frac{dy}{dx}\right|_{(1,\,2)} = 8$

An equation of the tangent line is $y = 2 + 8(x - 1)$.

$y = 8x - 8 + 2$

$y = 8x - 6$

$2: \begin{cases} 1: f'(1) \\ 1: \text{answer} \end{cases}$

(b) $f(1.1) \approx 2.8$

Since $y = f(x) > 0$ on the interval $1 \le x < 1.1$,

$\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2) > 0$ on this interval.

Therefore on the interval $1 < x < 1.1$, the line tangent to the graph of $y = f(x)$ at $x = 1$ lies below the curve and the approximation 2.8 is less than $f(1.1)$.

$2: \begin{cases} 1: \text{approximation} \\ 1: \text{conclusion with explanation} \end{cases}$

(c) $\frac{dy}{dx} = xy^3$

$\int \frac{1}{y^3}\,dy = \int x\,dx$

$-\frac{1}{2y^2} = \frac{x^2}{2} + C$

$-\frac{1}{2 \cdot 2^2} = \frac{1^2}{2} + C \Rightarrow C = -\frac{5}{8}$

$y^2 = \frac{1}{\frac{5}{4} - x^2}$

$f(x) = \frac{2}{\sqrt{5 - 4x^2}}, \quad \frac{-\sqrt{5}}{2} < x < \frac{\sqrt{5}}{2}$

$5: \begin{cases} 1: \text{separation of variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } y \end{cases}$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

2. The number of bacteria in a culture is increasing according to the law of exponential growth. After 5 hours there are 170 bacteria in the culture and after 10 hours there are 370 bacteria in the culture. Answer the following questions, rounding numerical answers to four decimal places.

(i) Find the initial population.

1 pt for K   1 pt for C   1 work

(ii) Write an exponential growth model for the bacteria population. Let t represent time in hours.

1 pt Model

(iii) Use the model to determine the number of bacteria after 20 hours.

1 pt for answer   1 pt for eqn

(iv) After how many hours will the bacteria count be 25,000?

1 Work   1 answer   1 equation

$(5, 170)$
$(10, 370)$

a) (i) 78.1081 ; (ii) $y = 78.1081e^{0.1555t}$; (iii) 1,752.6990 ; (iv) 37.0869 hr

b) (i) 80.2581 ; (ii) $y = 80.2581e^{0.2118t}$; (iii) 4,440.1503 ; (iv) 41.6494 hr

c) (i) 83.8881 ; (ii) $y = 83.8881e^{0.2189t}$; (iii) 7,191.6037 ; (iv) 43.7606 hr

d) (i) 85.4481 ; (ii) $y = 85.4481e^{0.2378t}$; (iii) 9,844.0672 ; (iv) 45.5400 hr

e) (i) 78.1081 ; (ii) $y = 78.1081e^{0.1767t}$; (iii) 3,065.4331 ; (iv) 39.4214 hr

Ans: a   Difficulty: Medium   Exercise Group: 64   Learning Objective: Create and solve exponential growth models   Section: 6.2   Type: Application

i) $y = Ce^{kt}$ Find C

$\boxed{C = 170\left(\frac{17}{37}\right) = 78.1081}$

$\frac{Ce^{5K}}{170} = \frac{Ce^{10K}}{370}$

$\frac{37}{17} = \frac{e^{10K}}{e^{5K}} = e^{5K}$

$K = \ln\left|\frac{37}{17}\right| / 5$

ii) $y = 78.1081e^{0.1555t}$

iii) $y = 78.1081e^{0.1555(20)} = 1752.699$

iv) $25000 = 78.1081e^{0.1555t}$

37.0869 hrs

FREE-RESPONSE QUESTION

A calculator may not be used on this question.

3. A differentiable function $f(x)$ is defined such that, for all values of x in its domain, $f(x) = 3 + \int_8^{x^3} f\left(\sqrt[3]{t}\right)\,dt$.

   a. What is the domain of $f(x)$?
   b. For what value(s) of x is $f(x) = 3$?
   c. Show that $f'(x) = 3x^2 f(x)$.
   d. Solve the differential equation in (c) to find $f(x)$ in terms of x only.

FREE RESPONSE

	Solution	Possible points
a.	$\sqrt[3]{t}$ is defined for all real numbers, so x^3 can be any real number; therefore x can be any real number.	1: answer
b.	$f(x) = 3$ when $x^3 = 8$; therefore $x = 2$ because the integral from 8 to 8 = 0.	1: answer
c.	$f'(x) = f\left(\sqrt[3]{x^3}\right)\cdot 3x^2$, so $f'(x) = 3x^2 f(x)$.	2: $\begin{cases} 1: \text{argument of } f(x) \\ 1: \text{Chain Rule} \end{cases}$
d.	Rewrite $f'(x) = 3x^2 f(x)$ as $\frac{dy}{dx} = 3x^2 y$. Separating variables, $\int \frac{dy}{y} = \int 3x^2\,dx$ $\ln\lvert y\rvert = x^3 + C$ $y = e^{x^3+C} = Ce^{x^3}$. Using $f(2) = 3$ (from answer b), we get $3 = Ce^8 \Rightarrow C = 3e^{-8}$ Therefore $y = 3e^{-8}e^{x^3}$, hence $f(x) = 3e^{x^3-8}$.	5: $\begin{cases} 1: \text{separation of variables} \\ 2: \text{correct antiderivatives} \\ 1: \text{includes constant of integration} \\ 1: \text{solves correctly for constant} \end{cases}$

$3 = 3 + \int_8^{x^3} f(\sqrt[3]{t})\,dt$

$3 = 3 + \int_8^{2^3} f(\sqrt[3]{t})\,dt$

$3 = 3 + 0$

$x = 2$

$f(x) = 3 + \frac{d}{dx}\int_8^{x^3} f(\sqrt[3]{t})\,dt$

$f'(x) = 0 + f(\sqrt[3]{x^3})(3x^2)$

$f'(x) = f(x)(3x^2)$

$= 3x^2 f(x)$

Graphing Calculator Allowed

1. Use integration to find a general solution of the differential equation.

$u = x^6$
$du = 6x^5\,dx$

$$\frac{dy}{dx} = x^5 e^{x^6}$$

$$\int dy = \int x^5 e^{x^6}\,dx$$
$$y = \frac{1}{6}\int 6x^5 e^{x^6}\,dx$$
$$y = \frac{1}{6}\int e^u\,du$$
$$\boxed{y = \frac{1}{6}e^{\cancel{x^6}\,u} + C}$$

a) $y = \frac{x^6}{6}e^{x^6} + C$

b) $y = 6e^{x^6} + C$

c) $y = \frac{1}{5}e^{x^6} + C$

d) $y = 5e^{x^6} + C$

e) $y = \frac{1}{6}e^{x^6} + C$

Ans: e Difficulty: Medium Exercise Group: 41-52 Learning Objective: Identify the general solution of a differential equation Section: 6.1 Type: Skill

2. Which of the following is a solution of the differential equation $xy' - 3y = x^4 e^x$?

$y' = 3x^2e^x + x^3e^x$

$\cancel{xy' - 3y = x^4e^x}$

$x(3x^2e^x + x^3e^x) - 3(x^3e^x) = x^4e^x$

$3x^3e^x + x^4e^x - 3x^3e^x = x^4e^x$

$0 = 0$

a) $y = x^3 e^x$

b) $y = 5e^{2x} - 6\sin 2x$

c) $y = 4e^{-2x}$

d) $y = \ln x$

e) $y = 3x^4 e^{2x}$

Ans: a Difficulty: Medium Exercise Group: 21-28 Learning Objective: Identify the solution of a differential equation Section: 6.1 Type: Skill

3. Which of the following is a solution of the differential equation $y^{(4)} - 256y = 0$?

a) $y = x^2(4 + e^x)$

b) $y = 3\ln x$

c) $y = e^{4x}$

d) $y = -4x\cos x$

e) $y = 2e^{4x} + x\sin x$

Ans: c Difficulty: Easy Exercise Group: 13-20 Learning Objective: Identify the solution of a differential equation Section: 6.1 Type: Skill

4. Find the particular solution of the differential equation $\frac{dr}{ds} = e^{r-2s}$ that satisfies the initial condition $r(0) = 0$.

a) $r = \ln(2 + e^{-2s}) + C$

b) $r = \ln(2) - \ln(1 + e^{-2s})$

c) $r = \ln\left|\frac{3 + e}{2}\right|$

d) [illegible]

e) [illegible]

$$\frac{dr}{ds} = \frac{e^r}{e^{2s}}$$
$$\frac{dr}{e^r} = \frac{ds}{e^{2s}}$$
$$-\int -e^{-r}\,dr = -\frac{1}{2}\int -2e^{-2s}\,ds$$
$$-e^{-r} + C_1 = -\frac{1}{2}e^{-2s} + C_2$$
$$-e^{-r} = -\frac{1}{2}e^{-2s} + C_{2-1}$$

$$2e^{-r} = e^{-2s} - 2C_{2-1}$$
$$\ln(2e^{-r}) = \ln(e^{-2s} - 2C_{2-1})$$
$$\ln 2 + \ln e^{-r} = \ln(e^{-2s} - 2C_{2-1})$$
$$\ln 2 - r = \ln(e^{-2s} - 2C_{2-1})$$
$$-r = -\ln 2 + \ln(e^{-2s} - 2C_{2-1})$$
$$r = \ln 2 - \ln(e^{-2s} - 2C_{2-1})$$

let $-2C_{2-1} = C$

C: $0 = \ln 2 - \ln(e^0 + C)$

$-\ln 2 = -\ln(1 - C)$

$e^{\ln 2} = e^{\ln(1+C)}$

$2 = 1 + C$

$1 = C$

$r = \ln 2 - \ln(e^{-2s} + 1)$

Ans: b Difficulty: Medium Exercise Group: 15-24 Learning Objective: Calculate the particular solution of the given differential equation with the given initial solution Section: 6.3 Type: Application

5. Find an equation of the graph that passes through the point (7,9) and has the slope $y' = \frac{8y}{9x}$.

a) $y = 9\left(\frac{x}{7}\right)^{[illegible]}$

b) $y = 9(7x)^{[illegible]}$

c) $y = 7\left(\frac{8x}{9}\right)^{9}$

d) $y = xe^{[illegible]}$

e) $y = \frac{9}{8x} - \ln(7x) + 9$

Ans: a Difficulty: Medium Exercise Group: 25-28 Learning Objective: Calculate the equation of a graph with a given slope and passing through the given point Section: 6.3 Type: Application

$$\frac{dy}{dx} = \frac{8y}{9x}$$

$$\frac{dy}{8y} = \frac{dx}{9x}$$

$$\frac{1}{8}\int \frac{dy}{y} = \frac{1}{9}\int \frac{dx}{x}$$

$$\frac{1}{8}\ln y + C_1 = \frac{1}{9}\ln x + C$$

$$\frac{1}{8}\ln y = \frac{1}{9}\ln x + C_2 -$$

6. Write and solve the differential equation that models the following verbal statement. Evaluate the solution at the specified value of the independent variable, rounding your answer to four decimal places:

The rate of change of y is proportional to y. When $x = 0$, $y = 56$ and when $x = 2$, $y = 72$. What is the value of y when $x = 10$?

a) $y(10) = 196.7480$

b) $y(10) = 200.4980$

c) $y(10) = 202.9580$

d) $y(10) = 192.9980$

e) $y(10) = 187.8280$

Ans: a Difficulty: Medium Exercise Group: 21-24 Learning Objective: Create and solve a differential equation model Section: 6.2 Type: Application

$$\frac{dy}{dx} = ky \qquad \int \frac{dy}{y} = \int k\,dx$$

$$\ln|y| = kx + C$$

$$C = \ln(56)$$

$$\ln|y| = kx + \ln(56)$$

$$\ln 72 = k(2) + \ln 56$$

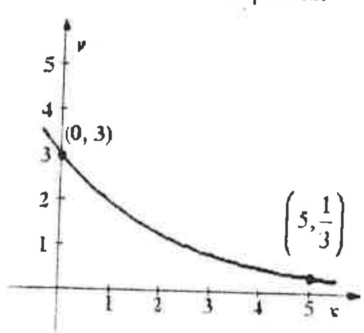
$$\frac{\ln(72) - \ln(56)}{2} = k = .12565$$

$$\ln y = .125657x + \ln 5$$

$$y = e^{.125657x + \ln 5}$$

$$\boxed{y = 196.74}$$

7. Find the exponential function $y = Ce^{kx}$ that passes through the two given points. Round your values of C and k to four decimal places.



a) $y = 3e^{1.8067x}$

b) $y = 3e^{0.1363x}$

c) $y = 3e^{0.2235x}$

d) $y = 3e^{0.9027x}$

e) $y = 3e^{0.4394x}$

Ans: e Difficulty: Easy Exercise Group: 25-28 Learning Objective: Identify an exponential equation given its graph Section: 6.2 Type: Skill

$$y = Ce^{kx}$$

$$3 = Ce^{0}$$

$$C = 3$$

$$\frac{1}{3} = 3e^{k(5)}$$

$$\frac{1}{9} = e^{5k}$$

$$\ln\left(\frac{1}{9}\right) = 5k$$

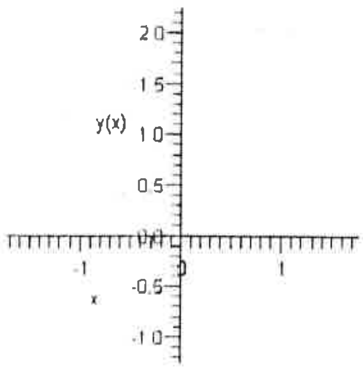
$$k = \frac{\ln\left(\frac{1}{9}\right)}{5}$$

$$k = -0.4394$$

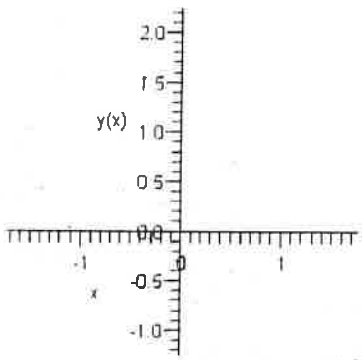
$$y = 3e^{-0.4394x}$$

8. Identify the slope field of the differential equation $\frac{dy}{dx} = e^{2x^2}$.

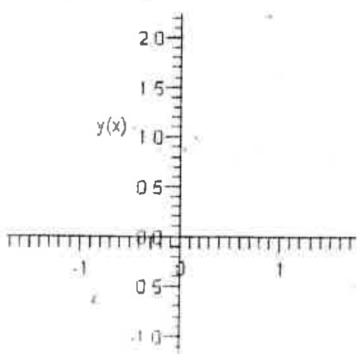
a)



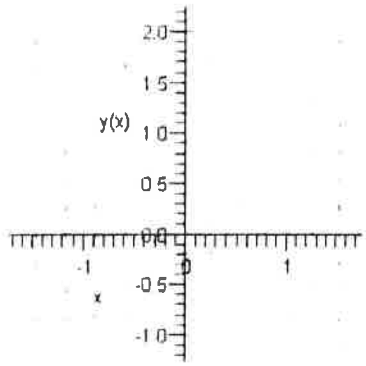
b)



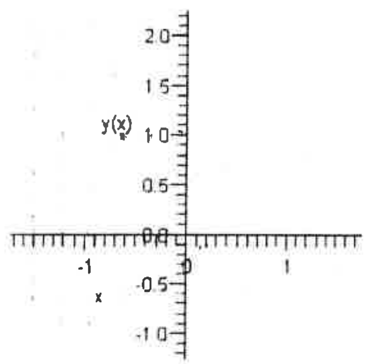
c)



d)



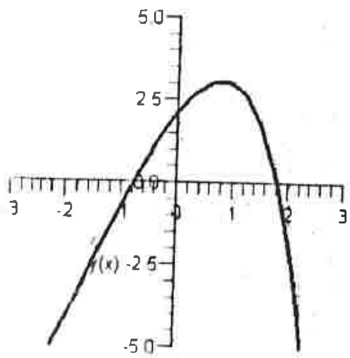
e)



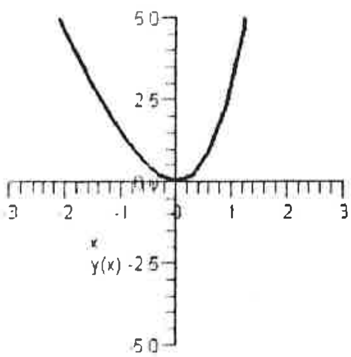
Ans: a   Difficulty: Medium   Exercise Group: 57-60   Learning Objective: Identify the slope field for a differential equation   Section: 6.1   Type: Skill

9. Sketch the slope field for the differential equation $y' = y - 4x$ and use the slope field to sketch the solution that passes through the point $(1,3)$.

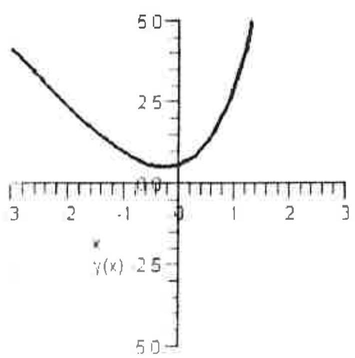
a)



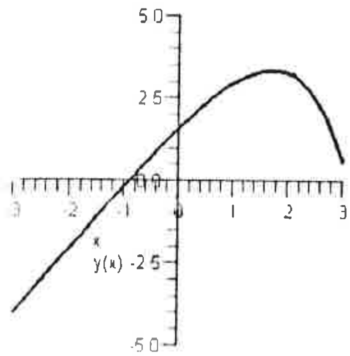
b)



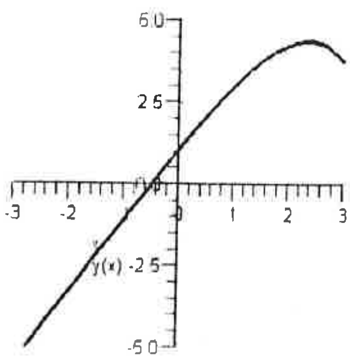
c)



d)



e)



Ans: a Difficulty: Easy Exercise Group: 61-64 Learning Objective: Identify the slope field for a differential equation and sketch a particular solution Section: 6.1 Type: Skill

10. The rate of change of N is proportional to N. When $t = 0$, $N = 180$ and when $t = 1$, $N = 480$. What is the value of N when $t = 2$? Round your answer to three decimal places.

a) 1,230.000
b) 1,280.000
c) 59.112
d) 1,310.000
e) 3,840.000

Ans: b Difficulty: Medium Exercise Group: 21-24 Learning Objective: Create and solve a differential equation model Section: 6.2 Type: Skill

11. The half-life of the carbon isotope C-14 is approximately 5715 years. If the initial quantity of the isotope is 40 g, what is the amount left after 5000 years? Round your answer to two decimal places.

a) 22.31 g
b) 17.00 g
c) 21.81 g
d) 29.54 g
e) 10.91 g

Ans: c *Difficulty: Easy* *Exercise Group: 33-40*
Learning Objective: Solve exponential growth/decay problems *Section: 6.2* *Type: Application*

12. The isotope ^{239}Pu has a half-life of 24,100 years. After 2,000 years, a sample of the isotope is reduced to 1.1 grams. What was the initial size of the sample (in grams)? How much will remain after 20,000 years (i.e., after another 18,000 years)? Round your answers to four decimal places.

a) 0.8156, 0.4588
b) 1.5147, 0.8521
c) 1.8642, 1.0488
d) 1.1651, 0.6555
e) 1.6312, 0.9177

Ans: d *Difficulty: Medium* *Exercise Group: 33-40*
Learning Objective: Solve exponential growth/decay problems *Section: 6.2* *Type: Application*

13. The initial investment in a savings account in which interest is compounded continuously is \$973. If the time required to double the amount is $7\frac{3}{4}$ years, what is the amount after 14 years? Round your answer to the nearest cent.

a) \$3403.37
b) \$3892.00
c) \$10,879.00
d) \$3515.35
e) \$3003.37

Ans: a *Difficulty: Easy* *Exercise Group: 43-48* *Learning Objective: Solve continuous compound interest problems* *Section: 6.2* *Type: Application*

14. A container of hot liquid is placed in a freezer that is kept at a constant temperature of 20° F. The initial temperature of the liquid is 180° F. After 5 minutes, the liquid's temperature is 68° F. How much longer will it take for its temperature to decrease to 34° F? Round your answer to two decimal places.

a) 3.07 minutes
b) 5.12 minutes
c) 4.61 minutes
d) 5.63 minutes
e) 2.05 minutes

Ans: b *Difficulty: Medium* *Exercise Group: 74*
Learning Objective: Solve exponential growth/decay problems *Section: 6.2* *Type: Application*

$$\frac{dT}{dt} = k(T-20)$$

(5, 68)
(0, 180)
(?, 34)

$$\ln(T-20) = kt + C$$
$$\ln(180-20) = k(0) + C$$
$$C = \ln(160)$$
$$\ln(68-20) = 5k + \ln(160)$$
$$k = \frac{\ln(48) - \ln(160)}{5} = -.240795$$

$$\ln(34-20) = t\left(\frac{\ln(48) - \ln(160)}{5}\right) + \ln(160)$$

$t = 10.117$
$- 5$ min
$= 5.117$ min

15. A calf that weighs 60 pounds at birth gains weight at the rate $\frac{dw}{dt} = k(1200 - w)$, where w is weight in pounds and t is time in years. If the animal is sold when its weight reaches 700 pounds, find the time of sale using the model $w = 1200 - 1140e^{-1.2t}$. Round your answer to two decimal places.

a) 0.69 year
b) 5.03 years
c) 2.05 years
d) 0.49 year
e) 1.47 years

$700 = 1200 - 1140e^{-1.2t}$

$t = .6868 = .69$ years

Ans: a *Difficulty: Medium* *Exercise Group: 63* *Learning Objective: Solve an exponential equation in applications* *Section: 6.3* *Type: Application*

16. A calf that weighs 55 pounds at birth gains weight at the rate $\frac{dw}{dt} = k(1100 - w)$, where w is weight in pounds and t is time in years. What is the maximum weight of the animal if one uses the model $w = 1100 - 1045e^{-1.1t}$?

a) 1045 lb
b) 655 lb
c) 1100 lb
d) 1700 lb
e) 1155 lb

Ans: c *Difficulty: Medium* *Exercise Group: 63* *Learning Objective: Calculate the relative extrema of a function in applications* *Section: 6.3* *Type: Application*

17. Solve the homogeneous differential equation $y' = \frac{9x + 11y}{x}$.

a) $y = \frac{9}{11}x + C \cdot x^{?}$
b) $y = -\frac{11}{8}x - x^{22} + C$
c) $y = -\frac{9}{10}x + C \cdot x^{11}$
d) $y = \frac{9}{10}x^{11} + C$
e) $y = \ln(9x + 11) + C$

Ans: c *Difficulty: Medium* *Exercise Group: 39-44* *Learning Objective: Calculate the given homogeneous differential equation* *Section: 6.3* *Type: Application*

$\frac{dy}{dx} = \frac{9x + 11y}{x} dx$

$y = Vx$

$dy = V + x\frac{dv}{dx}$

$V + x\frac{dV}{dx} = \frac{\cancel{9x} + 11(\cancel{Vx})}{\cancel{x}} dy$

$V + x\frac{dV}{dx} = 9 + 11V \, dx$

$x\frac{dV}{dx} = 9 + 11V - V$

$\frac{dV}{10V + 9} = \frac{dy}{x}$

$10 \ln|10V + 9| = \ln x + C$

$(10V + 9)^{10} = x + C$

$(10(\frac{y}{x}) + 9)^{10} = \cancel{8x} + C$

$10\frac{y}{x} + 9 = \sqrt[10]{x + K}$

$10\frac{y}{x} = \sqrt[10]{x + C} - 9$

$\frac{y}{x} = \frac{\sqrt[10]{x + C}}{10} - \frac{9}{10}$

$y = \frac{x\sqrt[10]{x + C}}{10} - \frac{9}{10}x$

Graphing Calculator Is Not Allowed!

1. Solve the differential equation $y' = \dfrac{\sqrt{x}}{2y}$.

a) $y = \sqrt{\frac{2}{3}x^{3/2} + C}$

b) $2y^2 = 4x^{3/2} + C$

c) $6y^2 = 2x^{3/2} + C$

d) $6y^2 = 4x^{3/2} + C$

e) $4y^2 = 6x^{3/2} + C$

Ans: d   Difficulty: Medium   Exercise Group: 1-10   Learning Objective: Identify the general solution of a differential equation   Section: 6.2   Type: Skill

2. Solve the differential equation.

$y' = x(-5 + y)$

a) $\ln|-5 + y| = x^2 + C$

b) $\ln|-5 + y| = x + C$

c) $3\ln|-5 + y| = x^3 + C$

d) $4\ln|-5 + y| = x^4 + C$

e) $2\ln|-5 + y| = x^2 + C$

$$\int \frac{dy}{-5+y} = \int x\,dx$$

$$\ln|-5+y| = \frac{x^2}{2} + C_1 \qquad 2C_1 = C$$

$$2\ln|-5+y| = x^2 + C$$

Ans: e   Difficulty: Medium   Exercise Group: 1-10   Learning Objective: Identify the general solution of a differential equation   Section: 6.2   Type: Skill

3. Solve the differential equation.

$y' = \dfrac{x}{y}$

a) $2\ln y = -x^3 + C$

b) $y^2 = x^3 + C$

c) $2\ln y = x^2 + C$

d) $y^2 = x^2 + C$

e) $y^2 = -x^2 + C$

Ans: d   Difficulty: Medium   Exercise Group: 1-10   Learning Objective: Identify the general solution of a differential equation   Section: 6.2   Type: Skill

4. Use integration to find a general solution of the differential equation.

$\dfrac{dy}{dx} = -2x^3 - 9x$

a) $2x^4 - 9x^2 + C$

b) $6x^2 - 9 + C$

c) $-\frac{1}{2}x^4 - \frac{9}{2}x^2 + C$

d) $2x^2 - 9 + C$

e) $\frac{1}{2}x^4 - \frac{9}{2}x^2 + x + C$

Ans: c   Difficulty: Medium   Exercise Group: 41-52   Learning Objective: Identify the general solution of a differential equation   Section: 6.1   Type: Skill

5. Use integration to find a general solution of the differential equation.

$\frac{dy}{dx} = -8x^2 - 7x$

$\int dy = \int -8x^2 - 7x \, dx$

$y = \frac{-8x^3}{3} - \frac{7x^2}{2} + C$

a) $y = 8x^2 - 7 + C$

b) $y = 8x^3 - 7x^2 + C$

c) $y = -16x - 7 + C$

d) $y = -\frac{8}{3}x^3 - \frac{7}{2}x^2 + C$

e) $y = -8x^2 - 7x + C$

Ans: d Difficulty: Medium Exercise Group: 41-52 Learning Objective: Identify the general solution of a differential equation Section: 6.1 Type: Skill

6. Use integration to find a general solution of the differential equation.

$\frac{dy}{dx} = x\sqrt{-x+5}$

$\int dy = \int x\sqrt{-x+5} \, dx$

$u = -x+5$

$du = -dx$

solve for x: $-x = u - 5$

$x = -u + 5$

$y = \int -x(u)^{1/2} du$

$y = \int -(-u+5)(u)^{1/2} du$

$y = \int (u-5)(u)^{1/2} du$

$y = \int u^{3/2} - 5u^{1/2} du$

$y = \frac{2(-x+5)^{5/2}}{5} - \frac{10}{3}(-x+5)^{3/2} + C$

a) $y = \frac{2}{5}(-x+5)^{5/2} - \frac{10}{3}(-x+5)^{3/2} + C$

b) $y = \frac{2}{5}(-x+5)^{3/2} - \frac{10}{3}(-x+5)^{5/2} + C$

c) $y = \frac{2}{5}(-x+5)^2 - \frac{10}{3}(-x+5) + C$

d) $y = (-x+5)^{3/2} - (-x+5)^{5/2} + C$

e) $y = \frac{2}{5}(-x+5)^{3/2} + \frac{10}{3}(-x+5)^{5/2} + C$

Ans: a Difficulty: Medium Exercise Group: 41-52 Learning Objective: Identify the general solution of a differential equation Section: 6.1 Type: Skill

7. Use integration to find a general solution of the differential equation $\frac{dy}{dx} = 5x\cos(15x^2)$.

$\int dy = \int 5x \cos(15x^2) dx$

$u = 15x^2$

$du = 30x \, d$

$y = \frac{1}{6}\int 6(5x)\cos(15x^2) dx$

$y = \frac{1}{6}\int \cos(u) du$

$y = \frac{1}{6} \sin(u) + C$

$y = 1/6 \sin(15x^2) + C$

a) $\frac{\sin(30x^2)}{3} + C$

b) $\frac{\sin(15x^2)}{6} + C$

c) $\frac{\cos(15x^2)}{6x} + C$

d) $\frac{2x\sin(30x^2)}{3x} + C$

e) $\frac{x\cos(15x^2)}{6} + C$

Ans: b Difficulty: Medium Exercise Group: 41-52 Learning Objective: Identify the general solution of a differential equation Section: 6.1 Type: Skill

Write and solve the differential equation that models the following verbal statement:

8. The rate of change of Q with respect to x is proportional to $40 - x$.

a) $\frac{dQ}{dx} = k(40-x)^{-1}$, $Q = -k\ln(40-x) + C$

b) $\frac{dQ}{dx} = k(40-x)$, $Q = -\frac{k}{2}(40-x)^2 + C$

c) $\frac{dQ}{dx} = k(40-x)^2$, $Q = -\frac{k}{3}(40-x)^3 + C$

d) $\frac{dQ}{dx} = k(40-x)^3$, $Q = -\frac{k}{4}(40-x)^4 + C$

e) $\frac{dQ}{dx} = k(40-x)^{-1}$, $Q = -k\ln(40-x)^2 + C$

Ans: b *Difficulty: Medium* *Exercise Group: 11-14* *Learning Objective: Create and solve a differential equation model* *Section: 6.2* *Type: Application*

9. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{9x^2}{10y^2}$.

a) $y = \frac{9}{10}x^3 + C$

b) $y = \sqrt[3]{\frac{10}{9}x^3 + C}$

c) $y = \sqrt[3]{9x^3 + 10C}$

d) $y = \sqrt[3]{\frac{9}{10}x^3 + C}$

e) $y = \sqrt[3]{\frac{x^3}{10} + C}$

Ans: d *Difficulty: Medium* *Exercise Group: 1-14* *Learning Objective: Calculate the general solution of the given differential equation* *Section: 6.3* *Type: Application*

10. Find the particular solution of the differential equation $19x + 4yy' = 0$ that satisfies the initial condition $y = 8$ when $x = 3$, where $19x^2 + 4y^2 = C$ is the general solution.

$19(3)^2 + 4(8)^2 = C$

$C = 427$

a) $19x^2 + 4y^2 = 1252$

b) $19x^2 + 4y^2 = 427$

c) $19x^2 + 4y^2 = 235$

d) $19x^2 + 4y^2 = 203$

e) $19x^2 + 4y^2 = 265$

Ans: b *Difficulty: Easy* *Exercise Group: 35-40* *Learning Objective: Identify the particular solution of a differential equation* *Section: 6.1* *Type: Skill*

11. Find the function $y = f(t)$ passing through the point $(0, 18)$ with the first derivative $\frac{dy}{dt} = \frac{2}{3}y$.

a) $y(t) = 18e^{\frac{2}{3}t^2}$

b) $y(t) = e^{\frac{2}{3}t^2} + 18$

c) $y(t) = \frac{2}{3}t^2 + 18$

d) $y(t) = 18e^{\frac{2}{3}t}$

e) $y(t) = e^{\frac{2}{3}t} + 18$

$\frac{dy}{y} = \frac{2}{3}dt$

$\ln y = \frac{2}{3}t + C$

$y\ Ce^{2/3t}$

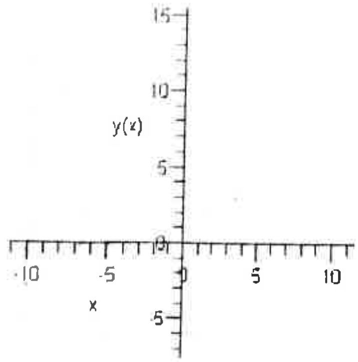
$18 = Ce^0$

$C = 18$

Ans: d *Difficulty: Medium* *Exercise Group: 17-20* *Learning Objective: Identify a function given its derivative and a point that it passes through* *Section: 6.2* *Type: Skill*

$y = 18e^{2/3t}$

12. Use the differential equation $\frac{dy}{dx} = \frac{3x}{y}$ and its slope field to find the slope at the point $(-5,5)$.



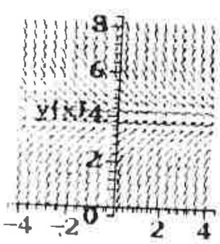
$$\frac{3(-5)}{5} = \frac{-15}{5} = \boxed{-3}$$

a) -75
b) -25
c) 15
d) -15
e) -3

Ans: e *Difficulty: Easy* *Exercise Group: 53-56* *Learning Objective: Evaluate a differential equation to find the slope of lines tangent to the solution of the differential equation* *Section: 6.1* *Type: Skill*

13. Sketch a few solutions of the differential equation on the slope field and then find the general solution analytically.

$$\frac{dy}{dx} = 4 - y$$



a) $y = C\ln(4-y)$
b) $y = 4 + Ce^{-x}$
c) $y = 4 - 2Ce^{x}$
d) $y = C\ln(y-4)$
e) $y = 4x + Ce^{x}$

Ans: b *Difficulty: Medium* *Exercise Group: 49-52* *Learning Objective: Solve a differential equation using separation of variables* *Section: 6.3* *Type: Skill*

4. Determine whether the function $f(x,y) = x^3 + 4x^2y^2 - 7y^3$ is homogeneous and determine its degree if it is.

a) homogeneous, the degree is 4
b) homogeneous, the degree is 5
c) homogeneous, the degree is 2
d) homogeneous, the degree is 3
e) not homogeneous

Ans: e *Difficulty: Easy* *Exercise Group: 31-38* *Learning Objective: Determine if the given function is homogeneous and find its degree* *Section: 6.3* *Type: Application*