

Semester One Final Review

Evaluate each limit.

1) $\lim_{x \rightarrow -1} -\frac{x^2 - 1}{x + 1}$

2

2) $\lim_{x \rightarrow -2} -\frac{x + 2}{x^2 - 4}$

 $\frac{1}{4}$

3) $\lim_{x \rightarrow -1} \frac{x + 1}{x^2 - x - 2}$

 $-\frac{1}{3}$

4) $\lim_{x \rightarrow 3} -\frac{x - 3}{x^2 - 2x - 3}$

 $-\frac{1}{4}$

5) $\lim_{x \rightarrow -1} (-2x + 1)$

3

6) $\lim_{x \rightarrow 0} 3$

3

7) $\lim_{x \rightarrow \frac{\pi}{4}} 2\tan(x)$

2

8) $\lim_{x \rightarrow \frac{\pi}{4}} -\cos(2x)$

0

9) $\lim_{x \rightarrow -\frac{5\pi}{6}} -\cot(2x)$
 $-\frac{\sqrt{3}}{3}$

10) $\lim_{x \rightarrow -1} \frac{x + 5}{x^2 - 7x + 10}$
 $\frac{2}{9}$

11) $\lim_{x \rightarrow 2} (x^3 - 3x^2 - 1)$
-5

12) $\lim_{x \rightarrow -2} (-2x + 3)$
7

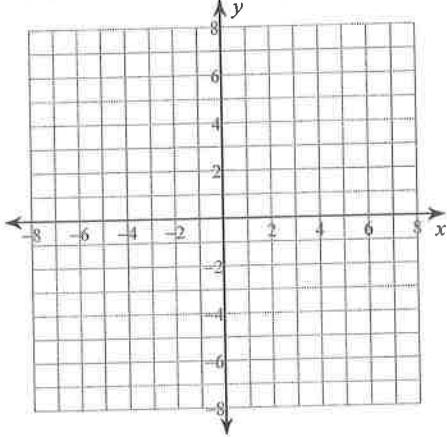
13) $\lim_{x \rightarrow -1^+} -\frac{x + 3}{x^2 + 4x + 3}$
-∞

14) $\lim_{x \rightarrow 2^-} \frac{x}{x - 2}$
-∞

15) $\lim_{x \rightarrow -\infty} \frac{x - 1}{x^2 + x + 1}$
0

16) $\lim_{x \rightarrow \infty} \frac{6}{x^2 + 3}$
0

17) Evaluate the following Limits.



Find the derivative of each function with respect to x .

$$18) \quad y = \sqrt{x+5}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+5}}$$

$$20) \quad y = 5x^2 - 4x + 1$$

$$\frac{dy}{dx} = 10x - 4$$

$$19) \quad y = \frac{1}{x+5}$$

$$\frac{dy}{dx} = -\frac{1}{x^2 + 10x + 25}$$

$$21) \quad y = -2x + 5$$

$$\frac{dy}{dx} = -2$$

Differentiate each function with respect to x .

$$22) \quad y = (2x^4 + 5)^2$$

$$\begin{aligned}\frac{dy}{dx} &= 2(2x^4 + 5) \cdot 8x^3 \\ &= 16x^3(2x^4 + 5)\end{aligned}$$

$$23) \quad y = (2x^2 + 3)^4$$

$$\begin{aligned}\frac{dy}{dx} &= 4(2x^2 + 3)^3 \cdot 4x \\ &= 16x(2x^2 + 3)^3\end{aligned}$$

24) $y = (5x^3 + 1)^2$

$$\begin{aligned}\frac{dy}{dx} &= 2(5x^3 + 1) \cdot 15x^2 \\ &= 30x^2(5x^3 + 1)\end{aligned}$$

25) $y = (2x + 1)^4$

$$\begin{aligned}\frac{dy}{dx} &= 4(2x + 1)^3 \cdot 2 \\ &= 8(2x + 1)^3\end{aligned}$$

For each problem, find the indicated derivative with respect to x .

26) $y = 3x + \sqrt[3]{x} + 4x^{-2}$ Find $\frac{d^3y}{dx^3}$

$$\frac{d^3y}{dx^3} = \frac{10}{27x^3} - \frac{96}{x^5}$$

27) $y = -3\sqrt[5]{x^2} + \sqrt[5]{x}$ Find $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = \frac{18}{25x^{\frac{8}{5}}} - \frac{4}{25x^{\frac{9}{5}}}$$

28) $y = 5x^5 + 5\sqrt[4]{x} + \sqrt[5]{x}$ Find $\frac{d^4y}{dx^4}$

$$\frac{d^4y}{dx^4} = \frac{600x}{256x^{\frac{15}{4}}} - \frac{504}{625x^{\frac{19}{5}}}$$

29) $y = x + \frac{2}{x^5}$ Find $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = \frac{60}{x^7}$$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ at the given point.

30) $-5y^2 + 4 = x^3$ at $(-1, 1)$

$$\left. \frac{dy}{dx} \right|_{\substack{x = -1 \\ y = 1}} = -\frac{3}{10}$$

31) $-5y^3 + 5 = 5x$ at $(2, -1)$

$$\left. \frac{dy}{dx} \right|_{\substack{x = 2 \\ y = -1}} = -\frac{1}{3}$$

For each problem, find the equation of the secant line that intersects the given points on the function and also find the equation of the tangent line to the function at the leftmost given point.

32) $y = 2x^2 - 2x + 1; (0, 1), \left(\frac{1}{3}, \frac{5}{9}\right)$

Secant: $y = -\frac{4}{3}x + 1$ Tangent: $y = -2x + 1$

33) $y = \frac{1}{x}; \left(2, \frac{1}{2}\right), \left(\frac{9}{4}, \frac{4}{9}\right)$

Secant: $y = -\frac{2}{9}x + \frac{17}{18}$ Tangent: $y = -\frac{1}{4}x + 1$

Differentiate each function with respect to x .

34) $y = \left(-3 + \frac{4}{x^4}\right)(2x^4 + 2x^2 - 3)$

$$\begin{aligned}\frac{dy}{dx} &= \left(-3 + 4x^{-4}\right)(8x^3 + 4x) + (2x^4 + 2x^2 - 3) \cdot -16x^{-5} \\ &= -24x^3 - 12x - \frac{16}{x^3} + \frac{48}{x^5}\end{aligned}$$

35) $y = \left(2 + \frac{2}{x^3}\right)(-5x^4 + x^3 + 1)$

$$\begin{aligned}\frac{dy}{dx} &= \left(2 + 2x^{-3}\right)(-20x^3 + 3x^2) + (-5x^4 + x^3 + 1) \cdot -6x^{-4} \\ &= -40x^3 + 6x^2 - 10 - \frac{6}{x^4}\end{aligned}$$

36) $y = \frac{5x^5 + x^4 + 2}{x^3 + 3}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x^3 + 3)(25x^4 + 4x^3) - (5x^5 + x^4 + 2) \cdot 3x^2}{(x^3 + 3)^2} \\ &= \frac{10x^7 + x^6 + 75x^4 + 12x^3 - 6x^2}{x^6 + 6x^3 + 9}\end{aligned}$$

$$37) \quad y = \frac{2x^5 + 1}{3x^5 + 2}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(3x^5 + 2) \cdot 10x^4 - (2x^5 + 1) \cdot 15x^4}{(3x^5 + 2)^2} \\ &= \frac{5x^4}{9x^{10} + 12x^5 + 4} \end{aligned}$$

$$39) \quad y = \cos(\sec 4x^4)$$

$$\begin{aligned} \frac{dy}{dx} &= -\sin(\sec 4x^4) \cdot \sec 4x^4 \tan 4x^4 \cdot 16x^3 \\ &= -16x^3 \sin(\sec 4x^4) \sec 4x^4 \tan 4x^4 \end{aligned}$$

$$40) \quad y = \sin(\cos 5x^3)$$

$$\begin{aligned} \frac{dy}{dx} &= \cos(\cos 5x^3) \cdot -\sin 5x^3 \cdot 15x^2 \\ &= -15x^2 \cos(\cos 5x^3) \sin 5x^3 \end{aligned}$$

$$41) \quad y = \frac{\tan 4x^3}{\sin 3x^4}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\sin 3x^4 \cdot \sec^2 4x^3 \cdot 12x^2 - \tan 4x^3 \cdot \cos 3x^4 \cdot 12x^3}{\sin^2 3x^4} \\ &= \frac{12x^2 (\sin 3x^4 \sec^2 4x^3 - x \tan 4x^3 \cos 3x^4)}{\sin^2 3x^4} \end{aligned}$$

$$38) \quad y = \tan(\sin 3x^4)$$

$$\begin{aligned} \frac{dy}{dx} &= \sec^2(\sin 3x^4) \cdot \cos 3x^4 \cdot 12x^3 \\ &= 12x^3 \sec^2(\sin 3x^4) \cos 3x^4 \end{aligned}$$

42) $y = \cot 4x^4 \sin x^3$

$$\begin{aligned}\frac{dy}{dx} &= \cot 4x^4 \cdot \cos x^3 \cdot 3x^2 + \sin x^3 \cdot -\csc^2 4x^4 \cdot 16x^3 \\ &= x^2(3\cot 4x^4 \cos x^3 - 16x \sin x^3 \csc^2 4x^4)\end{aligned}$$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ at the given point.

43) $5y^3 + 2 = 3x^3$ at $(-1, -1)$

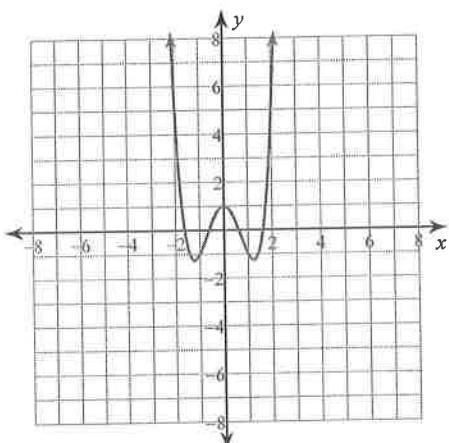
$$\left. \frac{dy}{dx} \right|_{\begin{array}{l} x = -1 \\ y = -1 \end{array}} = \frac{3}{5}$$

44) $-3x^3 + 3 = 3x + y^2$ at $(-1, 3)$

$$\left. \frac{dy}{dx} \right|_{\begin{array}{l} x = -1 \\ y = 3 \end{array}} = -2$$

For each problem, find the open intervals where the function is increasing and decreasing. You may use the provided graph to sketch the function.

45) $y = x^4 - 3x^2 + 1$



Increasing: $\left(-\frac{\sqrt{6}}{2}, 0\right), \left(\frac{\sqrt{6}}{2}, \infty\right)$ Decreasing: $\left(-\infty, -\frac{\sqrt{6}}{2}\right), \left(0, \frac{\sqrt{6}}{2}\right)$

For each problem, find all points of absolute minima and maxima on the given interval.

46) $y = -x^3 + 5x^2 - 7x + 1$; [0, 3]

Absolute minima: $(3, -2), (1, -2)$

Absolute maximum: $(0, 1)$

47) $y = (-6x + 30)^{\frac{1}{2}}$; [-2, 4]

Absolute minimum: $(4, \sqrt{6})$

Absolute maximum: $(-2, \sqrt{42})$

For each problem, find all points of relative minima and maxima.

48) $y = \frac{x^2}{2} - 4x + 8$

Relative minimum: $(4, 0)$

No relative maxima.

For each problem, find the values of c that satisfy the Mean Value Theorem.

49) $y = x^3 - 2x^2 + 1$; [0, 2]

$$\left\{ \frac{4}{3} \right\}$$

50) $y = x^3 + 2x^2 - 4x - 5$; [-3, 1]

$$\left\{ \frac{-2 + \sqrt{13}}{3}, \frac{-2 - \sqrt{13}}{3} \right\}$$

Solve each related rate problem.

- 51) A hypothetical square grows so that the length of its sides are increasing at a rate of 6 m/min. How fast is the area of the square increasing when the sides are 5 m each?

A = area of square s = length of sides t = time

Equation: $A = s^2$ Given rate: $\frac{ds}{dt} = 6$ Find: $\frac{dA}{dt} \Big|_{s=5}$

$$\frac{dA}{dt} \Big|_{s=5} = 2s \cdot \frac{ds}{dt} = 60 \text{ m}^2/\text{min}$$

- 52) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius of the spill increases at a rate of 9 m/min. How fast is the area of the spill increasing when the radius is 4 m?

A = area of circle r = radius t = time

$$\text{Equation: } A = \pi r^2 \quad \text{Given rate: } \frac{dr}{dt} = 9 \quad \text{Find: } \left. \frac{dA}{dt} \right|_{r=4}$$

$$\left. \frac{dA}{dt} \right|_{r=4} = 2\pi r \cdot \frac{dr}{dt} = 72\pi \text{ m}^2/\text{min}$$

For each problem, find the values of c that satisfy Rolle's Theorem.

53) $y = x^3 - 2x^2 - x - 1; [-1, 2]$

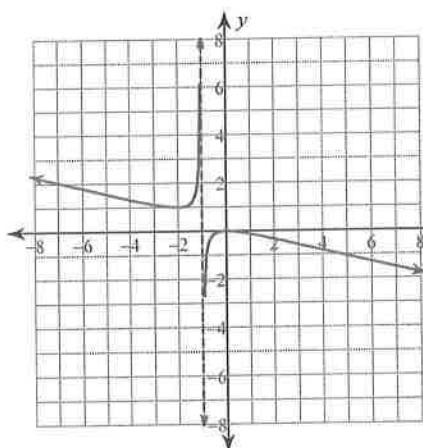
$$\left\{ \frac{2 + \sqrt{7}}{3}, \frac{2 - \sqrt{7}}{3} \right\}$$

54) $y = -x^3 + 3x^2 + x; [-1, 3]$

$$\left\{ \frac{3 - 2\sqrt{3}}{3}, \frac{3 + 2\sqrt{3}}{3} \right\}$$

For each problem, find the: x and y intercepts, asymptotes, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

55) $y = -\frac{x^2}{4x + 4}$



x-intercept at $x = 0$ y-intercept at $y = 0$

Vertical asymptote at: $x = -1$

No horizontal asymptotes exist.

$$\text{Slant asymptote: } y = -\frac{x}{4} + \frac{1}{4}$$

Critical points at: $x = -2, 0$

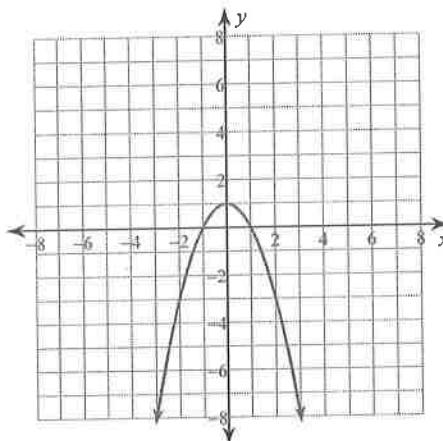
Increasing: $(-2, -1), (-1, 0)$ Decreasing: $(-\infty, -2), (0, \infty)$

No inflection points exist.

Concave up: $(-\infty, -1)$ Concave down: $(-1, \infty)$

Relative minimum: $(-2, 1)$ Relative maximum: $(0, 0)$

56) $y = -x^2 + 1$



x-intercepts at $x = -1, 1$ y-intercept at $y = 1$

No vertical asymptotes exist.

No horizontal asymptotes exist.

Critical point at: $x = 0$

Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$

No inflection points exist.

Concave up: No intervals exist. Concave down: $(-\infty, \infty)$

No relative minima. Relative maximum: $(0, 1)$

For each problem, find the differential dy .

57) $y = -x^3 - 1$

$$dy = -3x^2 dx$$

58) $y = \frac{2}{x}$

$$dy = -\frac{2}{x^2} dx$$

For each problem, find the general formulas for dy and Δy .

59) $y = -x^2 + 3$

$$dy = -2x dx$$

$$\Delta y = -2x \Delta x - (\Delta x)^2$$

60) $y = x^2 - 4x + 2$

$$dy = (2x - 4) dx$$

$$\Delta y = 2x \Delta x + (\Delta x)^2 - 4 \Delta x$$

For each problem, find dy and Δy , given x_0 and $dx = \Delta x$.

61) $y = \frac{3}{x}; x_0 = -1, dx = \Delta x = -\frac{3}{2}$

$$dy = \frac{9}{2} = 4.5$$

$$\Delta y = \frac{9}{5} = 1.8$$

62) $y = x^2 + 4x + 2; x_0 = -4, dx = \Delta x = 1$

$$dy = -4$$

$$\Delta y = -3$$