

Year End Final Review

Evaluate each limit.

1) $\lim_{x \rightarrow 1^+} \frac{x-2}{x^2 - 3x + 2}$
 ∞

2) $\lim_{x \rightarrow 3^+} -\frac{3}{x^2 - 9}$
 $-\infty$

3) $\lim_{x \rightarrow -\infty} (-2x^2 - 16x - 33)$
 $-\infty$

4) $\lim_{x \rightarrow \infty} \left(\frac{\cos x}{x} + 4 \right)$
 4

5) $\lim_{x \rightarrow 1^-} \lfloor 2x + 2 \rfloor$
 3

6) $\lim_{x \rightarrow 0^-} f(x), f(x) = \begin{cases} -2x - 3, & x < 0 \\ \frac{x}{2} - 3, & x \geq 0 \end{cases}$
 -3

7) $\lim_{x \rightarrow 2} -\frac{x-2}{x^2 - x - 2}$
 $-\frac{1}{3}$

8) $\lim_{x \rightarrow 0} \frac{\sin(2x)}{\sin(4x)}$
 $\frac{1}{2}$

9) $\lim_{x \rightarrow 1} -2x$
 -2

10) $\lim_{x \rightarrow 0} \frac{x+2}{x^2 + 6x + 9}$
 $\frac{2}{9}$

11) $\lim_{h \rightarrow 0} \frac{\left(\frac{1}{2} + h\right)^2 - \left(\frac{1}{2}\right)^2}{h}$
 1

12) $\lim_{h \rightarrow 0} \frac{\left(-\frac{1}{2} + h\right)^3 + \frac{1}{8}}{h}$
 $\frac{3}{4}$

Evaluate each sum.

13) $\sum_{k=1}^n (k^2 + 3)$
 $\frac{n^3}{3} + \frac{n^2}{2} + \frac{19n}{6}$

14) $\sum_{k=1}^n (2k^2 + 6)$
 $\frac{2n^3}{3} + n^2 + \frac{19n}{3}$

Differentiate each function with respect to x .

15) $y = (3x^4 + 1)^2 \quad \frac{dy}{dx} = 2(3x^4 + 1) \cdot 12x^3$
 $= 24x^3(3x^4 + 1)$

16) $y = ((x^4 + 3)^{-5} - 2)^{-2} \quad \frac{dy}{dx} = -2((x^4 + 3)^{-5} - 2)^{-3} \cdot -5(x^3)$
 $= \frac{40x^3(x^4 + 3)^9}{(-2(x^4 + 3)^5 + 1)^3}$

Use the definition of the derivative to find the derivative of each function with respect to x .

17) $y = -4x + 5$

$$\frac{dy}{dx} = -4$$

18) $y = \frac{1}{2x + 5}$

$$\frac{dy}{dx} = -\frac{2}{4x^2 + 20x + 25}$$

For each problem, find the indicated derivative with respect to x .

19) $y = 5x^5$ Find $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = 100x^3$$

20) $y = x^4 + 3x^2 + 5x$ Find $\frac{d^4y}{dx^4}$

$$\frac{d^4y}{dx^4} = 24$$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

21) $5x + 4y^3 = 1$

$$\frac{dy}{dx} = -\frac{5}{12y^2}$$

22) $2 = 5x^2 + 5xy^3 + x^2y^3$

$$\frac{dy}{dx} = \frac{10x + 5y^3 + 2xy^3}{-15xy^2 - 3x^2y^2}$$

For each problem, find the instantaneous rate of change of the function at the given value.

23) $y = 2x^2 + x - 2$; -1

$$-3$$

For each problem, find the equation of the secant line that intersects the given points on the function and also find the equation of the tangent line to the function at the leftmost given point.

24) $y = -x^2 - 2$; $(1, -3)$, $\left(\frac{4}{3}, -\frac{34}{9}\right)$

Secant: $y = -\frac{7}{3}x - \frac{2}{3}$ Tangent: $y = -2x - 1$

For each problem, find $(f^{-1})'(x)$

25) $f(x) = 2x + 5$

$$(f^{-1})'(x) = \frac{1}{2}$$

26) $f(x) = 2x^7 + 3x + 4$

$$(f^{-1})'(x) = \frac{1}{14y^6 + 3}, \text{ where } y = f^{-1}(x)$$

Differentiate each function with respect to x .

27) $y = \cos^{-1} 3x^2$ $\frac{dy}{dx} = -\frac{1}{\sqrt{1 - (3x^2)^2}} \cdot 6x$

$$= -\frac{6x}{\sqrt{1 - 9x^4}}$$

28) $y = \cot^{-1} \sqrt[5]{4x^3 - 5}$

$$\begin{aligned} \frac{dy}{dx} &= -\frac{1}{\left((4x^3 - 5)^{\frac{1}{5}}\right)^2 + 1} \cdot \frac{1}{5}(4x^3 - 5) \\ &= -\frac{12x^2}{5\sqrt[5]{(4x^3 - 5)^4}(\sqrt[5]{(4x^3 - 5)^2} + 1)} \end{aligned}$$

Use logarithmic differentiation to differentiate each function with respect to x .

29) $y = 4x^{x^3}$

$$\frac{dy}{dx} = y(3x^2 \ln x + x^2)$$

Differentiate each function with respect to x .

30) $y = e^{5x^4}$

$$\frac{dy}{dx} = e^{5x^4} \cdot 20x^3$$

32) $y = 4x^5$

$$\frac{dy}{dx} = 20x^4$$

34) $y = (5x^4 - 2) \cdot -3x^2$

$$\begin{aligned}\frac{dy}{dx} &= (5x^4 - 2) \cdot -6x - 3x^2 \cdot 20x^3 \\ &= -90x^5 + 12x\end{aligned}$$

36) $y = \frac{2}{5x^3 - 3}$ $\frac{dy}{dx} = -\frac{2 \cdot 15x^2}{(5x^3 - 3)^2}$
 $= -\frac{30x^2}{25x^6 - 30x^3 + 9}$

31) $y = \log_4 x^2$ $\frac{dy}{dx} = \frac{1}{x^2 \ln 4} \cdot 2x$
 $= \frac{2}{x \ln 4}$

33) $y = 4x^5 + \frac{5}{4}x^{\frac{5}{2}} + 5x^{\frac{5}{4}}$

$$\frac{dy}{dx} = 20x^4 + \frac{25x^{\frac{3}{2}}}{8} + \frac{25x^{\frac{1}{4}}}{4}$$

35) $y = (-3 - 5x^{-5})(-2x^2 + 4)$ $\frac{dy}{dx} = (-3 - 5x^{-5}) \cdot -4x + (-2x^2 + 4) \cdot 5x^{-6}$
 $= 12x - \frac{30}{x^4} + \frac{100}{x^6}$

37) $y = \frac{3x^5 - 2x^4}{5x^{\frac{4}{5}} + 5}$ $\frac{dy}{dx} = \frac{\left(5x^{\frac{4}{5}} + 5\right)(15x^4 - 8x^3) - (3x^5 - 2x^4)}{\left(5x^{\frac{4}{5}} + 5\right)^2}$
 $= \frac{63x^{\frac{24}{5}} + 75x^4 - 32x^{\frac{19}{5}} - 40x^3}{8x^{\frac{4}{5}}}$

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$, and their derivatives. Use the table data and the rules of differentiation to solve each problem.

38)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-2	2	1
2	1	$-\frac{1}{2}$	3	1
3	2	1	4	$-\frac{1}{2}$
4	3	1	2	-2

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(2)$ $h_1'(2) = \frac{1}{2}$
 Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(1)$ $h_2'(1) = -3$

Differentiate each function with respect to x .

39) $y = \sin 2x^3$

$$\begin{aligned}\frac{dy}{dx} &= \cos 2x^3 \cdot 6x^2 \\ &= 6x^2 \cos 2x^3\end{aligned}$$

40) $y = \sec(2x^3 + 1)^3$

$$\begin{aligned}\frac{dy}{dx} &= \sec(2x^3 + 1)^3 \tan(2x^3 + 1)^3 \cdot 3(2 \\ &= 18x^2 \sec(2x^3 + 1)^3 \tan(2x^3 + 1)^3\end{aligned}$$

For each problem, find all points of absolute minima and maxima on the given interval.

41) $y = -x^2 + 6x - 4; [3, 6]$

Absolute minimum: $(6, -4)$

Absolute maximum: $(3, 5)$

Evaluate each definite integral.

42) $\int_2^4 (x^3 - 4x^2 + 6) dx$

$$-\frac{8}{3} \approx -2.667$$

43) $\int_{-1}^0 -\frac{3}{(x-2)^3} dx$

$$\frac{5}{24} \approx 0.208$$

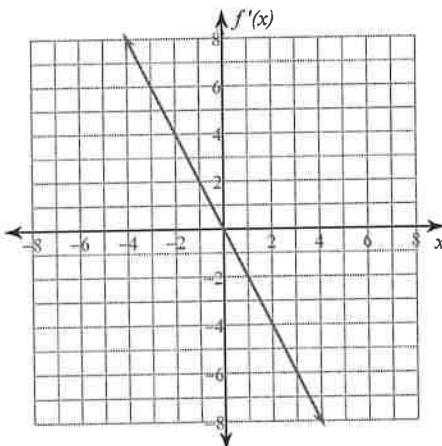
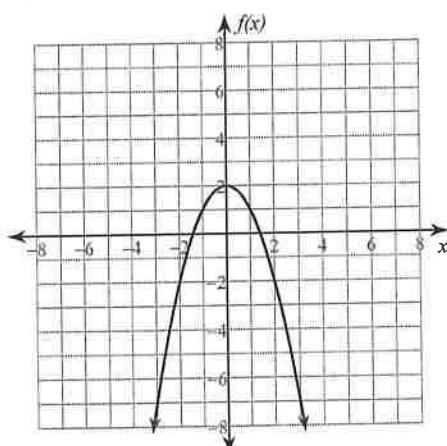
For each problem, find $F'(x)$.

44) $F(x) = \int_{-5}^x (t^2 + 6t + 9) dt$

$$F'(x) = x^2 + 6x + 9$$

Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$.

45)



For each problem, find the indicated derivative with respect to x .

46) $y = 3x^5 + x$ Find $\frac{d^3y}{dx^3}$

$$\frac{d^3y}{dx^3} = 180x^2$$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

47) $x + 2y^2 + 3y = 2$

$$\frac{dy}{dx} = -\frac{1}{4y+3}$$

Evaluate each limit. Use L'Hôpital's Rule if it can be applied. If it cannot be applied, evaluate using another method and write a * next to your answer.

48) $\lim_{x \rightarrow \infty} \frac{x}{\ln x}$

∞

For each problem, find the average value of the function over the given interval.

49) $f(x) = -2x^2 - 12x - 16; [-5, -1]$

$$-\frac{2}{3} \approx -0.667$$

For each problem, find the values of c that satisfy the Mean Value Theorem.

50) $y = x^2 + 6x + 4; [-5, -1]$

$$\{-3\}$$

Express each definite integral in terms of u , but do not evaluate.

51) $\int_0^2 \frac{8x}{(2x^2 + 2)^2} dx; u = 2x^2 + 2$

$$\int_2^{10} \frac{2}{u^2} du$$

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the x -axis.

52) $y = \sqrt{x+3}, y = 0, x = -2, x = 3$ $\pi \int_{-2}^3 (\sqrt{x+3})^2 dx$ $y = \sqrt{x}, y = 0, x = 4$ $\pi \int_0^4 (\sqrt{x})^2 dx$
 $= \frac{35}{2}\pi \approx 54.978$ $= 8\pi \approx 25.133$

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the given axis.

54) $y = \sqrt{x} + 4, y = x^2 + 4$ Axis: $y = -1$ $\pi \int_0^1 ((\sqrt{x} + 5)^2 - (x^2 + 5)^2) dy$ $y = x^2 - 1, y = \sqrt{x} - 1$ Axis: $y = 2$
 $= \frac{109}{30}\pi \approx 11.414$ $\pi \int_0^1 ((3 - x^2)^2 - (3 - \sqrt{x})^2) dx$
 $= \frac{17}{10}\pi \approx 5.341$

For each problem, find all points of absolute minima and maxima on the given interval.

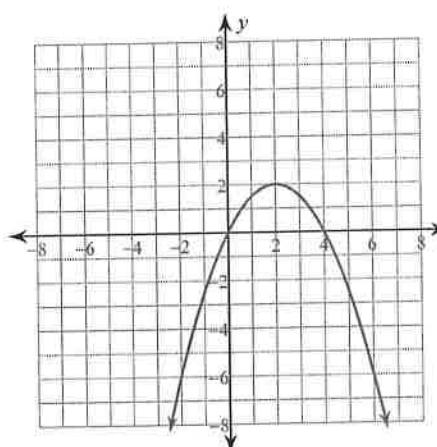
56) $y = -2x^2 - 12x - 12; [-4, -2]$

Absolute minima: $(-4, 4), (-2, 4)$

Absolute maximum: $(-3, 6)$

For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

57) $y = -\frac{x^2}{2} + 2x$



x-intercepts at $x = 0, 4$ y-intercept at $y = 0$

Critical point at: $x = 2$

Increasing: $(-\infty, 2)$ Decreasing: $(2, \infty)$

No inflection points exist.

Concave up: No intervals exist. Concave down: $(-\infty, \infty)$

No relative minima. Relative maximum: $(2, 2)$

For each problem, find the values of c that satisfy Rolle's Theorem.

58) $y = x^3 - 3x^2 - x + 2; [-1, 3]$

$$\left\{ \frac{3+2\sqrt{3}}{3}, \frac{3-2\sqrt{3}}{3} \right\}$$

For each problem, find the area of the region enclosed by the curves.

59) $y = \frac{3}{x^2}, y = -4,$

$x = -3, x = -1$

$$\int_{-3}^{-1} \left(\frac{3}{x^2} + 4 \right) dx \\ = 10$$

60) $y = \frac{x^3}{2} + \frac{x^2}{2} - \frac{5x}{2}, y = \frac{x}{2}$

$$\int_{-3}^0 \left(\frac{x^3}{2} + \frac{x^2}{2} - \frac{5x}{2} - \frac{x}{2} \right) dx \\ \int_0^2 \left(\frac{x}{2} - \left(\frac{x^3}{2} + \frac{x^2}{2} - \frac{5x}{2} \right) \right) dx \\ = \frac{253}{24} \approx 10.542$$